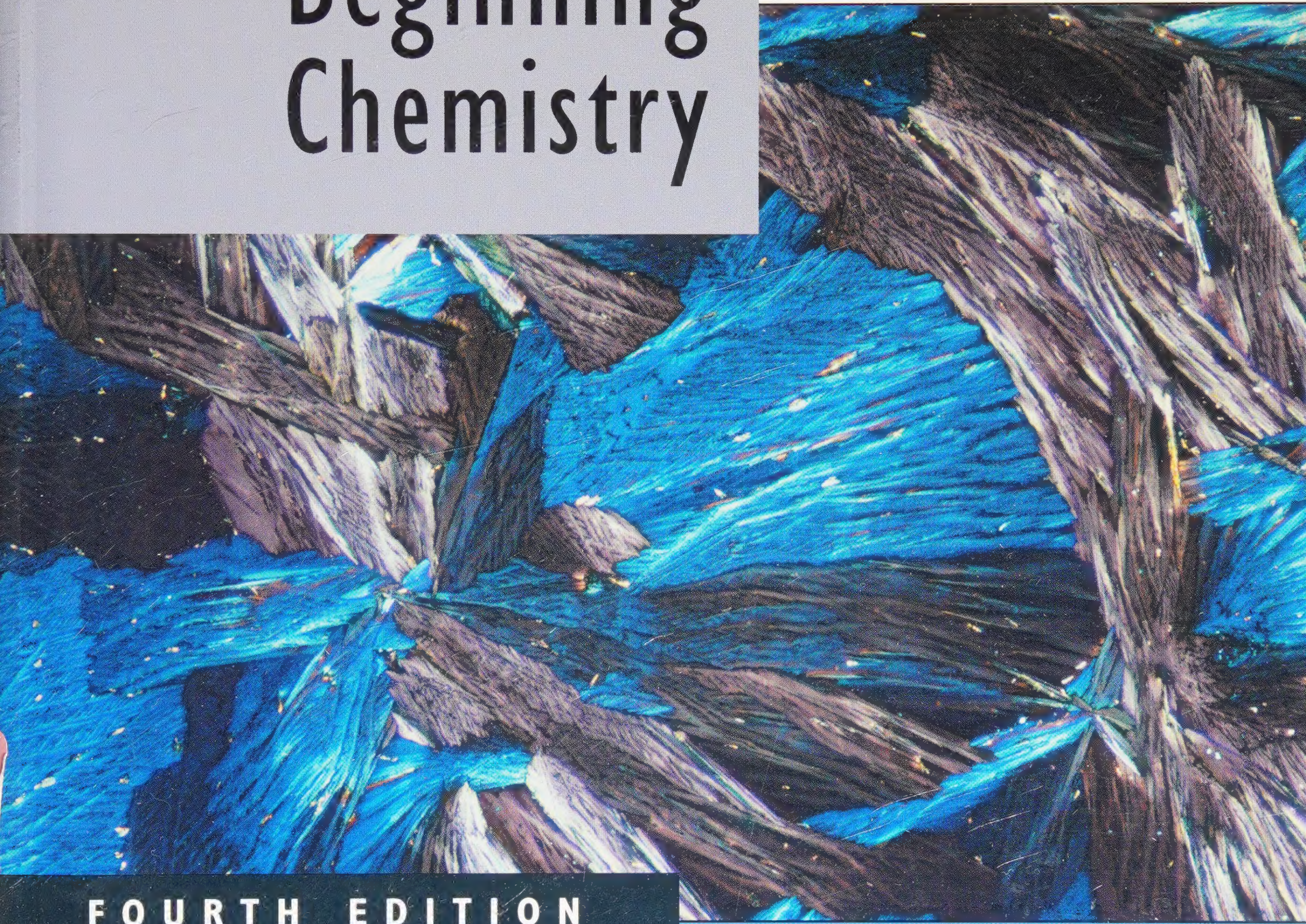
Basic Mathematics FOR Beginning Chemistry

DOROTHY M. GOLDISH



FOURTH EDITION

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To

Sivie and Irving
for their continuing love and support

and to

Elihu, Judy, and Matt
for their patience and assistance

TO THE STUDENT

Although this book is entitled *Basic Mathematics for Beginning Chemistry*, it is neither a mathematics textbook nor a chemistry text. Instead, it is intended to help you to refresh your memory of mathematics and to gain experience in applying the mathematics to work in chemistry.

This book is especially for students who consider themselves "no good at math" but who are taking a science course. Does that describe you? If so, you are probably rather scared because you have always heard that science uses a lot of mathematics (although you haven't the slightest idea why—or maybe you think that the reason is to keep anyone but the initiated from understanding what mysterious things these people are doing). It is no help that the instructor claims that elementary algebra is all that will be used. You passed algebra but . . . but you don't remember it, or you didn't understand it—you just followed some incomprehensible rules.

Why are you "no good at mathematics"? Maybe it's the way your mind happens to work or maybe it's the result of your early mathematics classes. Perhaps your teacher one year was "no good at math" and managed to convey this attitude to the class. Perhaps you were so bored with the drill required to learn arithmetic that you refused to think seriously about any other branch of mathematics. Perhaps you didn't learn the material one year. Lacking that year's material, you had trouble the following years. Whatever the reason, early problems with arithmetic need not close the world of mathematics to you forever.

Use whatever chapters and sections of the book are helpful to *you*. Do you need suggestions for methods that help you solve problems? Do you

need to understand why you carry out the processes needed to solve problems? Do you need to be able to handle graphs and equations with understanding? Do you just need plenty of practice, so that you can do the mathematics without worrying? It is my hope that this book will help you gain for yourself both the skills and the confidence needed to succeed in your chemistry course. Good luck!

D.M.G.

CONTENTS

CHAPTER 1

GETTING STARTED

1.1. INTRODUCTION

This book is intended to provide practice with a variety of mathematical skills used in beginning chemistry courses at various levels.

The first two chapters present some very fundamental material, because some students have difficulties that stem from their failure to understand these basic concepts. Skip material you already know, and work with the sections that are useful to you. Although material covered in Chapters 1–4 is used throughout the book, Chapters 5–8 can be studied in whatever order you find useful. The last two chapters contain material that may be useful to students in a general chemistry course, but might not be required in other beginning courses.

The problems in the book use everyday examples, numbers, letters, and chemical terms. Although definitions for many of the chemical terms are given, no attempt has been made to teach the chemistry involved. These examples are used merely to provide practice in using the calculation methods. You should refer to your chemistry textbook for discussion of the chemical concepts.

Some problems are marked with an asterisk; the solutions to these problems are given in detail at the end of the chapter. Answers to all problems are given at the end of the book.

1.2. CALCULATORS

With electronic calculators readily available at low prices, most chemistry instructors expect students to use calculators on problems and examinations. The availability of calculators has taken much of the fear

out of calculations for most students. However, using a calculator does not guarantee correct answers. You must still set up the problem correctly, enter the calculations correctly, and interpret the result.

If you are planning to buy a calculator to use in your chemistry class, your instructor may have advice on what type to buy. It is highly desirable to have a scientific calculator, which has exponents, exponential notation (marked EE or EXP), and logarithm (LOG) functions. Many calculators do not have these functions. Even if your calculator has them, it is important that you understand what you are trying to do and what sort of answer to expect.

Some students choose calculators with far more advanced capabilities than they need, perhaps hoping that the calculator will take some of the work out of the course. Unfortunately, such calculators are not only not helpful, they may be a hindrance because they may be harder to use.

Once you have purchased a calculator, take the time to get acquainted with *your* calculator. There are two entirely different ways of entering data and operations on calculators. Each model has its own features. Therefore, it is essential to spend some time reading the instruction book and practicing calculations.

Try adding a few numbers, such as 1 + 2 + 3. Try some multiplication problems, such as 2 x 3 and 2(8)/4. Try correcting an error; punch in 2 + 6, then change the 6 to a 4. For all of these, you can easily compute the result mentally so that you can check the result obtained on the calculator.

If your calculator has additional functions, learn how they work. Some students learn how to use a few of the built-in functions and never have time to learn the others; this may give satisfactory results, but it loses its efficiency. For example, you can find a square root by pushing a square root key; by using INV y^x keys; or by finding the logarithm, dividing by two, then finding the antilog. All give the same result, but each uses a different set of calculator functions and takes a different length of time.

When you complete any calculation, check your answer. Do not assume it is correct, just because it appears on the calculator display.

Does the answer make sense?
Does the answer contain the right number of significant figures?
Have you finished the problem?

A wrong answer may be caused by reading the problem incorrectly, setting up the problem incorrectly, or by pushing the wrong keys, thus entering data or operations incorrectly. You can check your answer two ways. Form the habit of doing a rapid mental calculation (Chapter 4) for all problems, to estimate the approximate answer. Compare the answer with the picture you draw to see if the size makes sense. Reread

the original question, to see if you have calculated the quantity required. Often students are so pleased at accomplishing a difficult calculation successfully that they forget to complete a final step.

A word about computers: Even if you have access to a personal computer, you may not find it helpful for beginning chemistry calculations. For most work, a calculator is faster and more convenient to use. A computer is likely to be convenient only when you must repeat a calculation for many sets of numbers.

The same device that applies to calculators applies to work with computers. Learn how to enter the commands and data, and practice until you can do it accurately without having to take time to look up the rules every time. One of the most maddening things about computers is that they do what they are told, not what you mean to tell them. Furthermore, they do what you command only if you enter the commands in precisely the required way. Be careful about such details as punctuation and exact abbreviations. Every program or programming language has its own specific requirements. Take care to enter numbers accurately. Check the answer to see if it makes sense.

1.3. LOOKING AT PROBLEMS

Most work in chemistry is not only descriptive but also quantitative. That is, it deals not only with what happens, but also with questions of "how much?" This book is intended to help you work a variety of quantitative problems. Some advice:

1. If your teacher prefers a method that is different from the one in this book, use your teacher's method. If your teacher prefers a method of rounding off or of writing significant figures that is different from the one in this book, you may need to correct the answers at the end to reflect this alternate method.
2. There is some basic information about notations and general rules of mathematics in Chapter 2 and Section 10.1. If any of this is unfamiliar to you, or if you are not sure you remember the rules correctly, work through that material before you do anything else.
3. For all kinds of problems, you can help yourself succeed by doing the following:

Draw a picture.
Write it down.
Think about your answer.

Most chemistry problems require several steps. Even if the calculation can be done in one step, you may need to gather several pieces of information before you can start. Don't try to keep everything in your head. Every time you find information you need, and every time you figure out

a step in solving the problem, write it down. Write down anything you plan to enter on your calculator. Write down any answer. The time needed to write down clearly labeled information will, in the end, be made up by time saved understanding what you are doing. You don't need to use your best handwriting, but your work must be organized and clearly labeled, so you can use it without having to think it through again.

Before we discuss ways to draw pictures to illustrate problems, let's review the metric system of measurements.

1.4. METRIC SYSTEM

For scientific work, measurements are made in the metric system, the system of measurements used in most countries in the world. The advantage of this system is that units are related by powers of 10 so that conversion to units of different sizes requires only moving a decimal point.

Prefixes

For all measurements in the metric system there is some unit that is used without a prefix on the name. Smaller and larger units are designated by prefixes (unlike English units, where the names as well as the numerical relationships are arbitrary). The prefix for 1000 of the basic units is *kilo-*, abbreviated k. The prefix abbreviation is combined with the unit abbreviation so that kilogram is written kg. For a smaller unit, 1/1000 as large as the fundamental unit, the prefix *milli-*, abbreviated m, is used. It is occasionally useful to use a unit 1/100 as big as the fundamental unit; this has the prefix *centi-*, abbreviated c. For even smaller units, 1/1,000,000 of the fundamental unit has the prefix *micro-*, abbreviated with the Greek letter μ (mu); 1/1,000,000,000 of the fundamental unit has the prefix *nano-*, abbreviated n. No periods are used after the abbreviations for the units. The metric units are summarized in Table 1.1. To convert from one unit to another requires only moving the decimal point the correct number of places (often three). A quantity of a given size may be measured by a few large units or many small ones, just as a given amount of money might be paid with 1 dollar bill, with 10 dimes, or with 100 pennies. The larger the size of each one unit, the more of them. Thus 2 m = 2000 mm (not 0.002 mm) since there must be a lot of the small units, mm, to make up the 2 large meters. Verify this by looking at a ruler and by doing the calculation by dimensional analysis (Chapter 5).

TABLE 1.1 METRIC UNITS*

Prefix	*Size*		*Mass*	*Length*	*Volume*
mega, M	1,000,000 units	$= 10^6$	*megagram*, Mg	*megameter*, Mm	*megaliter*, ML
kilo, k	1000 units	$= 10^3$	kilogram, kg	kilometer, km	*kiloliter*, kL
	unit		gram, g	meter, m	liter, L or l
deci, d	$\frac{1}{10}$ unit	$= 10^{-1}$	*decigram*, dg	*decimeter*, dm	*deciliter*, dL
centi, c	$\frac{1}{100}$ unit	$= 10^{-2}$	*centigram*, cg	centimeter, cm	*centiliter*, cL
milli, m	$\frac{1}{1000}$ unit	$= 10^{-3}$	milligram, mg	millimeter, mm	milliliter, mL
micro, μ	$\frac{1}{1,000,000}$ unit	$= 10^{-6}$	microgram, μg	micrometer, μm (micron)	microliter, λ (lambda)
nano, n	$\frac{1}{1,000,000,000}$ unit	$= 10^{-9}$	nanogram, ng	nanometer, nm	*nanoliter*, nL

* Italicized units are seldom used.

EXAMPLE 1

An object has a mass of 25 g. What is its mass in (a) kg and (b) mg?

For each, you must move the decimal point three places, as indicated by the exponent 3 in Table 1.1. At first glance, there seems to be no way to move the decimal point in either direction, since there are not three digits before or after the original position of the decimal point. You do not change the size of a number if you add additional zeros before or after a number as placeholders, as long as you are careful not to place any zeros between the original number and the original position of the decimal point. Write in the zeros; then move the decimal.

A kilogram equals 1000 g, so there must be a smaller number associated with the unit kilogram than with the unit gram. The decimal must be moved three places to the left:

$$25 \text{ g} = 0.025. \text{g} = 0.025 \text{ kg}$$

A milligram is much smaller than a gram, so there must be many milligrams for each gram. Therefore, the decimal point must be moved three places to the right.

$$25.000. \text{g} = 25{,}000. \text{ mg}$$

Length

Length is a measurement of distance or extension. The metric unit of length is the meter,* which measures about 39.37 inches, a little longer than 1 yard. The meter is now officially defined as 1,650,763.73 wavelengths in vacuum of a specific radiation of the krypton-86 atom. There are instruments that use krypton radiation for measurement, but for everyday use it is obviously more common and convenient to measure by comparison with a calibrated bar. Examples of such bars are rulers and meter sticks.

Although the meter is a convenient unit for many purposes, from measuring fabrics to measuring distances at track meets, it is rather small for measuring distances between cities; for this the kilometer (km) is used. A kilometer is about 0.6 mile. For a great deal of other work the centimeter (cm) is more useful than the very small millimeter (mm). One inch is 2.54 cm or 1 cm = 0.4 in.

Mass

Mass is the quantity that most people mean when they refer to weight. Mass refers to the amount of a substance present and is measured by the procedure referred to (inconsistently) as weighing. (Weight is a measure of the effect of gravity on a given mass. The mass of a substance does not change, but the weight depends on the force of gravity.)

Since the gram is inconveniently small for everyday use, the kilogram is commonly used. The legal standard of mass is the standard kilogram, a carefully preserved piece of metal with which other masses can be compared. One kilogram is about 2.2 lb.

Volume

Volume is not a fundamental measurement because it can be defined in terms of length. However, it is convenient (as well as traditional) to have a defined unit of volume with which to work. The unit used in the metric system is the liter, which is about 1.06 quarts. Various definitions have been used, but the liter is now defined as the volume of a cube 1 decimeter (dm) or 0.1 m on a side, or as 1000 milliliters (mL), where 1 mL is the volume of a cube 1 cm on a side.

Note: This means that 1 mL = 1 cm^3. Furthermore, many scientists speak interchangeably of milliliters and cubic centimeters.

* The people who devised the metric system, in France, wished to develop a rational and interrelated system of units, based on some universal measurement. They chose as the unit of length, the meter, one ten-millionth of the distance from the North Pole to the Equator on the meridian passing through Paris. For convenience in making measurements, this distance was marked on a metal bar, with which other measuring sticks could be compared. A unit of mass was defined in terms relating to the length of the meter by making use of that ubiquitous liquid, water. One gram was defined as the mass of 1 cubic centimeter of water.

Time

Time, along with mass and length, is a fundamental unit, that is, one that is not defined in terms of the other units. The standard unit of time is the second.

Temperature

Temperature measurements in science use the *Celsius* scale (formerly called centigrade) or the *Kelvin* (*absolute*) scale. The Celsius scale is based on the use of the freezing point of water as 0° and the normal boiling point of water as 100°. Celsius temperatures are related to Fahrenheit temperatures by the following equation:

$$\frac{°\mathrm{C}}{°\mathrm{F} - 32°} = \frac{5}{9}$$

The Kelvin temperature is obtained by adding 273° (more precisely, 273.15°) to the Celsius temperature. The advantage of the Kelvin scale is that all temperatures are positive. Modern usage omits the degree sign from Kelvin temperatures, so 25°C would be written 298 K.

Energy

In the international system of units (SI units), the energy unit is the joule, abbreviated J. The joule is defined in terms of the fundamental metric units.

$$1 \text{ joule} = 1 \text{ kg} \cdot \text{m/sec}^2$$

Older reference works use a unit, the calorie, that is defined as the heat needed to raise the temperature of 1 gram of water by 1 degree Celsius or, more specifically, from 14.5°C to 15.5°C. (The calorie used in measuring the energy available from foods is the kilocalorie, sometimes indicated by spelling "calorie" with a capital C.)

$$1 \text{ cal} = 4.184 \text{ J}$$

Another unit, the erg, is defined as 10^{-7} joule.

PROBLEMS†

1.1 What is the full name of each unit?

(a) cm (b) mm (c) nm
(d) kg (e) mL (f) mg

† Solutions to the starred problems are given at the end of each chapter.

1.2 Convert from the unit given to the one indicated, for each problem.

(a) 0.25 m to cm *(b) 0.25 m to mm
(c) 372 mg to g (d) 0.293 L to mL
*(e) 29 g to kg (f) 2.0 kg to g
(g) 2932 m to km

1.3 Perform the unit conversions indicated.

*(a) 525 mL to L *(b) 525 mL to cm^3
(c) 1.32 L to mL (d) 1.32 L to cm^3

1.5. DRAW A PICTURE

Students often feel lost when they try to deal with quantities expressed in scientific units, even when they can do the same calculation easily if it is expressed in familiar units. Why can you do a calculation involving apples or gumdrops or inches, and then not be able to do the same calculation when it involves moles or nanometers? There are two answers. The first is selling yourself short. You tell yourself that you don't know the answer, so you don't try to calculate it. The second is that you can't picture what the problem means and don't understand the words.

When you watch a lesson presented on television, the teacher shows you a picture and refers to the picture repeatedly. If you are studying or working problems, you will often find that drawing your own picture helps you to understand a concept or evaluate an answer. It is also helpful to think about real-life examples and to look at objects you can handle such as rulers or molecular models.

Another common problem is that words used in science are sometimes ones that are also used for other things. It's bad enough when a word means nothing to you, but it gets even more confusing when the mental picture called up by a word doesn't fit anything that makes sense. You might be sure that a "mole" as used by a chemist is not a little furry animal that makes a mess of lawns, or a dark spot on the skin, but sometimes it's not so obvious that there's a problem. It can be hard to understand a definition. Sometimes the only way to grasp the meaning is to use the definition repeatedly. If you have to use words you don't understand to solve problems you don't understand, where do you start? Start by drawing a picture. The picture doesn't have to be artistically drawn or detailed; a general scheme that helps you understand the information is enough.

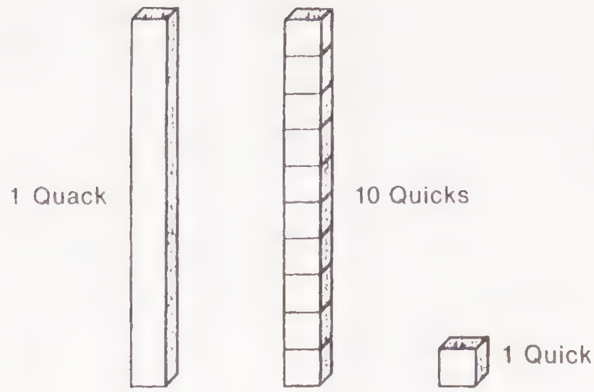


FIGURE 1.1 Ten quicks are the same size as (equal to) 1 quack.

EXAMPLE 2

You are asked to calculate the number of quacks in 57 quicks. The units *quick* and *quack* are defined in one of the following ways. All these definitions say the same thing; they are just phrased differently.

A quack contains 10 quicks.

A quick is 0.1 quack.

There are 10 quicks in 1 quack.

Start by drawing a picture of the relative size of the units (Figure 1.1). The names of the units are, like many units in science, very similar as well as being words with other meanings. They are unfamiliar to you, of course, since they are not real units; they were made up for purposes of this problem.

Now the units look "real." You can see that the number of quicks is much larger than the number of quacks, for the same-size item. This helps you check the result of your calculation. If you calculate that 57 quicks are 5.7 quacks, that is reasonable; the number of quacks must be smaller than the number of quicks, since it takes a pile of 10 quicks to make up 1 quack. If you did the calculation wrong and came up with an answer that 57 quicks equal 570 quacks, you could see that the answer is wrong; 570 quacks would have to be an awful lot of quicks. ■

Many relationships in scientific work are ratio units. That is, they are quantities that show the relationship between two other units. You encounter many ratio units in everyday life. You drive your car at a speed of 40 miles per hour (40 miles traveled for every 1 hour moving at that speed). Food and beverage containers are labeled to show the number of servings per container.

Such ratio units are harder to illustrate, because you must somehow show two units combined into one. Probably the best way to show such a relationship is to draw the two units side by side, enclosed in a box to indicate they are part of the same overall unit.

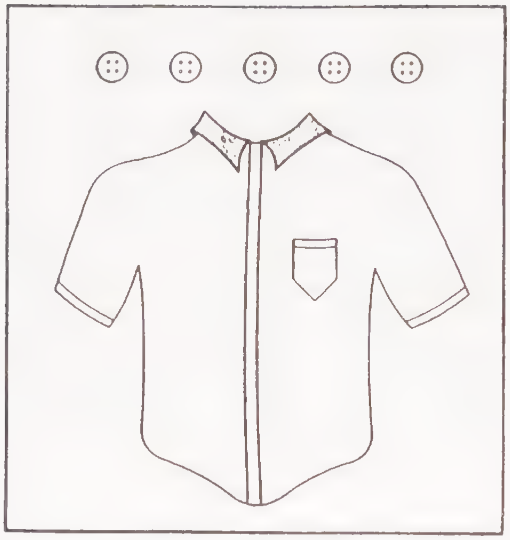


FIGURE 1.2 Five buttons for every one shirt.

It may help to illustrate this concept with familiar objects before we try to draw a picture of a ratio unit used in chemistry.

■ EXAMPLE 3

You are buying buttons for shirts. (Assume you can't find new buttons that match the old ones, so that you are buying all new buttons.) If each shirt has five buttons, you are working with a ratio unit, five buttons/shirt. You need an illustration showing a grouping of five buttons for every one shirt (Figure 1.2). ■

■ EXAMPLE 4

The density of silver is 10.5 g/cm^3. The density of octane (a component of gasoline) is 0.70 g/cm^3. You are asked to calculate the mass of 50 cm^3 of each.

Density is another ratio unit. Here, the first question is how to draw an indication of the units. It is easy to draw a unit of volume, such as 1 cm^3 by drawing a picture of a cube 1 cm on a side (Figure 1.3). If you recognize that 1 cm^3 = 1 mL, you can draw a container with a substance up to the 1-mL mark; this might make you more comfortable than drawing a cube when you are talking about a liquid. ■

How can you show the number of grams associated with 1 cm^3? A gram is a unit of mass (the quantity you commonly think of as weight). You could draw a balance (a "scale") with the pointer pointing to the mass, but that wouldn't be much help when you need to judge whether

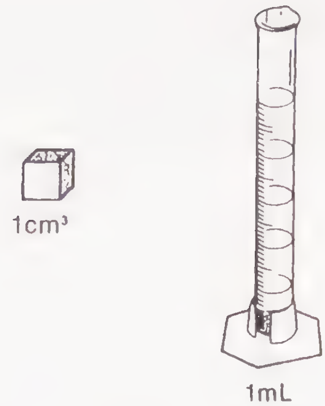
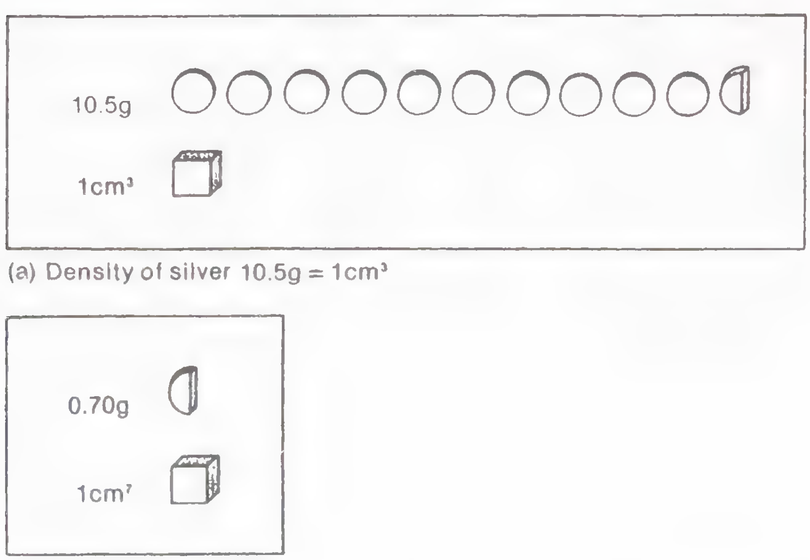


FIGURE 1.3

the result of a calculation is a reasonable answer. It is more helpful to draw objects you can count. Traditionally, the objects used have been old-fashioned weights, metal cylinders with narrow necks. Until recent times, substances to be weighed were placed on one pan of a balance, and objects of known mass ("weights") were placed on the other pan, until the balance was level. The mass of the object then equaled the sum of the masses of the weights used.

Use whatever objects you find convenient to represent the mass, perhaps coins or stones. The important thing is to use something that helps *you* understand the relationship.

Figure 1.4 shows you that the number of grams of silver is much bigger than the number of cm^3. For octane, the number of grams is



(a) Density of silver $10.5g = 1cm^3$

(b) Density of octane $0.70g = 1cm^3$

FIGURE 1.4

smaller than the number of cm^3. When you have carried out the calculation in the problem, you should check to see that your answer fits this pattern. For silver, 50 cm^3 = 535 g. If you made a mistake and got 54.7 g or 0.21 g (answers that would result from two common errors in the setup), you should be able to see that the answer is wrong. In the first instance, 54.7 g is a number that is almost the same size as 50 cm^3, but the picture in Figure 1.4 shows that the number of grams must be a great deal bigger than the number of cm^3. An answer like 0.21 g cannot be correct, since the number of grams cannot be smaller than number of cm^3. Similarly, if you get the answer that 50 cm^3 of octane have a mass of 35 g, you can see that the answer is reasonable. If you calculated that the mass was 71 g, the answer must be wrong, since the number of grams of octane must be smaller than the number of cm^3.

PROBLEM

1.4 One mole of carbon is defined as the amount of carbon that has a mass of 12 grams. Use an illustration to help you answer the questions.
 (a) How many grams are there in 2 moles of carbon?
 (b) How many grams are there in 0.5 mole (half a mole) of carbon?
 (c) You calculate that 15 grams of carbon are 180 moles. Is your answer correct?
 (d) On a multiple-choice exam, a question asks for the number of grams in 2.5 moles of carbon. Without doing the calculation, select the answer most likely to be correct:
 (1) 0.21 g (2) 3.0 g (3) 30 g (4) 3000 g

It can be helpful to draw pictures to help you understand words and concepts, even when you are not working a problem. For example, let's say you find the words used for the different parts of a solution are really confusing, since they all sound alike. Again, draw pictures. A *solute* dissolves in a *solvent* to form a *solution*. The solute may be a solid, liquid, or gas. The solvent is most often a liquid. Draw a picture of a solid solute and a liquid solvent, forming a liquid solution. Label each carefully.

It also helps to see a real example. You probably have a chance to do that in the laboratory, but you can easily find real-life examples. Put a spoonful of sugar (the solute) into a glass. Add water (the solvent) and stir to make a solution. The solution is clear, since it is homogeneous; that is, it is all a liquid, with no solid particles to block the light. It is colorless, as is usual when both solvent and solute are colorless.

Form the habit of drawing pictures whenever you need help visualizing the size or relationship of measurements or of objects. For most

problems, the picture can help you both to understand the calculation and to judge whether your answer is reasonable.

A word of caution: An incorrect picture can be misleading. Once you visualize an incorrect picture of a concept, it can be hard to correct your thinking.

1.6. SIGNIFICANT FIGURES

Measurements, and calculations based on measurements, must be reported to the correct number of significant figures. Significant figures are those that convey information. If you report that 8913 people attended an event, the reader is justified in expecting that you counted the number of people present carefully and know that the number was exactly 8913 and not some other number, such as 8920 or 8910. On the other hand, if you wrote that the attendance was about 9000, the reader would know that you had a general idea about the number of people present but had not made a precise count. In the first number, all four digits are significant, since all four convey information about the count. In the figure 9000, it is not at all clear how many digits are significant, since the zeros are needed as placeholders; there is no indication of the precision of the number.

When a number represents a count of objects, it is an exact number. Five apples are exactly five and not a little under or over. However, numbers representing measurements are not similarly exact. Measurements are limited in precision by the measuring instrument used. If you are weighing or measuring a ball, you would not need to weigh or measure very carefully if the ball is a toy. You would need better measuring instruments to make sure a ball to be used in professional competition meets the legal standards.

If you are measuring an object with a ruler marked in centimeters, you can say that the object is nearer the 5-cm mark than any other. You can estimate that it is not as much as one tenth of a centimeter to either side of that mark so that it is larger than 4.9 cm and smaller than 5.1 cm. It would then be correct to indicate that the length of the object is 5.0 cm. Such an indication means that you know the 5 exactly but are in a little doubt about the second figure, with a variation of 0.1 either way (unless a larger variation is indicated). It would not be correct to write the length as 5.00 cm, since with some doubt about the second figure you know nothing whatsoever about the third. The measurement 5.0 cm is said to have two *significant figures*.

A measurement should be written in such a way that the last digit shown, but only the last digit, is in doubt. For instance, if you use a burette in the laboratory, the 23.2- and 23.3-mL marks are indicated. Therefore, you know the first place after the decimal with certainty and

can estimate the next place. Hence you can report a volume reading as 23.29 mL with four significant figures. You should not, however, indicate a measurement as 23.200 mL, with five significant figures, even if it appears to be right on the mark; the markings on the instrument leave the second decimal place in some doubt so that nothing is known about the third. Equally, a liquid level right at the 5.0-mL mark should be recorded as 5.00 mL, not as 5 mL. A reported reading of 5 mL would indicate that the 5 is in some doubt.

1.6.A. Determining the Number of Significant Figures in a Number

To determine the number of significant figures in a number, count all digits that are not zero and any zeros that come between non-zero digits. There would be four significant figures in the number 8003. Count zeros written to the right of a decimal point when they occur *after* other (non-zero) digits. Do not count zeros that are used only to position the decimal point. The number 2.00 has three significant figures; the zeros after the 2 are not needed to show the position of the decimal point and are included to show that the first decimal place is known exactly and the second is in doubt. The number 2.00 can be taken to indicate that the value is known to be greater than 1.99 and less than 2.01. On the other hand, the number 0.002 has one significant figure; zeros come before the number and are needed to locate the correct position of the decimal.

EXAMPLE 5

How many significant figures are present in each number?

(a) 1.0062 (b) 0.0062 (c) 0.00620 (d) 106.20

All figures in 1.0062 are significant, since the zeros come between non-zero digits. It has five significant figures.

For 0.0062 the zeros are used only to position the decimal point and are therefore not counted. There are two significant figures.

The number 0.00620 has two kinds of zeros. Those before the 6 are needed to show the position of the decimal point and are not significant figures. The one after the 2, however, is not needed to locate the decimal; it is present to show that that place is known with some accuracy. There are therefore three significant figures.

In 106.20, there are five significant figures. One zero comes between two non-zero digits. The second zero is at the right of the decimal point and after non-zero digits. ■

When zeros come after other digits but to the left of the decimal point, there is considerable ambiguity about the number of significant figures.

The number 720 might have either two or three significant figures. This ambiguity is removed if the number is written in exponential notation (see Chapter 3). If there are two significant figures, the number would be written 7.2×10^2, and if there are three significant figures, the number would be written 7.20×10^2. If no clear indication is given, use the number 720 as if it had three significant figures; extra figures can always be dropped later.

1.6.B. Use of Significant Figures in Calculations

It is important to report the result of a calculation in a way that reflects correctly the precision of the original experimental measurement. If you have measured the dimensions of a rectangular solid as 28 cm by 22 cm by 16 cm (all measurements to two significant figures), it is meaningless to report that the volume of the solid is 9856 cm^3, even though the product of $28 \times 22 \times 16$ is indeed 9856. The number 9856 has four significant figures and implies that the volume is known to be between 9855 and 9857 cm^3. This is incorrect, since the volume is not known with that degree of accuracy. A variation of 1 in 9856 is a possible error of 0.01%. In fact, the possible error is 1 in 16 (the least precise measurement), or 6%. The result reported can never be more accurate than the measurement on which it is based. The volume should have been reported as 9900 cm^3 or 9.9×10^3 cm^3, rounded off to two significant figures.

To round off, drop all the extra figures. If the first figure dropped is 4 or less, the result is your answer. If the first figure dropped is 5 or more, add 1 to the last figure remaining

$$9.746 \rightarrow 9.7$$

$$9.756 \rightarrow 9.8$$

Some people prefer a modification of this rule. If the first figure dropped is 5, round to the even number. In other words, if the first figure dropped is 5, look at the last figure remaining. If that figure is odd, add 1 to the last figure remaining. If that figure is even, do not add 1.

$$9.75 \rightarrow 9.8$$

$$9.85 \rightarrow 9.8$$

Your calculator is no help in determining the appropriate number of significant figures in an answer. It will show far too many digits in most calculations, but will omit final zeros that may, in fact, be significant. **You must determine the number of significant figures that is appropriate in your answer** and round off or add final zeros to show that number. If

your calculator gives an answer of 5 and your answer should have three significant figures, write the answer as 5.00.

The number of significant figures in the answer should match the least precise measurement. There are some general rules that help you determine this.

When adding or subtracting measurements, round off to the decimal place of the least precise measurement. This is the measurement having the fewest decimal places, not the one with the fewest significant figures.

If you measure a distance as 29 cm, this means that the length is between 28 cm and 30 cm, since there is a possible variation of 1 in the last significant figure. If you then add a length of 0.05 cm to the original measurement, the total will still be 29 cm within the limitations of your measuring device.

EXAMPLE 6

Add 125 cm, 5.39 cm, and 32.905 cm.

The measurement made with least precision is 125 cm, with a possible variation of 1 cm. Therefore, the measurements should be rounded off to the ones column. Some people round off before adding, on the basis that it is not possible to add something to an empty column. Others prefer to add first, then round off.

$$\begin{array}{r@{\;=\;}r} 125 & 125 \text{ cm} \\ 5.39 & 5 \text{ cm} \\ \underline{32.905} & \underline{33 \text{ cm}} \\ 163.295 & 163 \text{ cm} \end{array}$$

This answer implies a doubt in the last place shown, that is, a possible variation of 1 cm, exactly as in the least precise measurement. ■

When multiplying or dividing measurements, report the result of the calculation to the same number of significant figures as the measurement having the fewest significant figures. It is necessary to use the count of significant figures, rather than the position of the decimal point, since the position of the decimal point changes in these calculations.

EXAMPLE 7

Multiply 451 mm × 36 mm and report the result to the correct number of significant figures.

The least precise measurement in the data is 36 mm, with two significant figures. Therefore, the product must be shown to two significant figures.

$$451 \text{ mm} \times 36 \text{ mm} = 1.6 \times 10^4 \text{ mm}^2 \text{ or } 16{,}000 \text{ mm}^2$$

It would not be correct to write the answer as 16,236 mm^2. ■

■ **EXAMPLE 8**

Perform the calculation and report the answer to the correct number of significant figures.

$$\frac{1.250 \text{ cm} \times 4800 \text{ cm}}{7.50 \text{ cm}}$$

The least precise measurement is 7.50 cm, with three significant figures. Therefore, the answer must be shown to three significant figures. The answer is 0.800 cm, *not* 0.8 cm. ■

Although this procedure is the one used in most general chemistry books, and is therefore the one that will be used in this book, some people prefer to retain one additional significant figure when the first digit is 1. Using this method, the answer to Example 7 would be reported as 1.62×10^4 mm^2.

PROBLEMS

1.5 How many significant figures are there in each measurement?
(a) 51 cm (b) 1.0023 g (c) 0.005 g
(d) 0.0210 m (e) 40.00 mL

1.6 Report each sum to the correct number of significant figures.
(a) 150 g + 2.39 g + 0.012 g
(b) 0.137 g + 0.0022 g + 0.011 g
(c) 12 mL + 0.12 mL + 0.012 mL

1.7 Report the result of each calculation to the correct number of significant figures.

(a) $37 \times 92 \times 1.297$ (b) $\dfrac{25 \times 720}{5.2}$

(c) $\dfrac{9.56 \times 0.0500}{0.239}$ (d) $25.0 \times 4 \times 2.88$

SOLUTIONS TO STARRED PROBLEMS

1.2(b) To convert m to mm, the decimal must be moved three places to the right (many small units, mm, equal few large units, m). There are only two digits shown, so a zero must be added to provide the third place.

$$0.25 \text{ m} = 0.250 \text{ m}$$

$$= 250 \text{ mm}$$

(e) To convert g to kg, move the decimal three places to the left (a few big kg equal many smaller g. Here, again, a zero must be provided as a placeholder.

$$29 \text{ g} = 029 \text{ g}$$

$$= .029 \text{ kg} \quad \text{or} \quad 0.029 \text{ kg}$$

(A zero is usually written before the decimal to help show that a decimal point is there.)

1.3(a) To convert from mL to L, move the decimal three places to the left.

$$520 \text{ mL} = 0.520 \text{ L}$$

(b) Since mL = cm^3, 525 mL = 525 cm^3.

CHAPTER 2

REVIEW OF MATHEMATICAL PROCEDURES

There are some procedures and rules of arithmetic and algebra used repeatedly in work in science that everyone assumes you know. In fact, students frequently get tripped up because they don't really remember some of the fundamentals.

Many people are not sure what they are doing when they must use positive and negative numbers or percents or ratios. Most people feel somewhat insecure in working with fractions. This chapter reviews notations and the rules for various calculations. If you are really certain of your knowledge of these fundamentals, you can skip Sections 2.1–2.5. Sections 2.6–2.10 are likely to be a useful review for most students.

2.1. NOTATIONS

2.1.A. Use of Letters in Equations

Mathematical rules are commonly expressed in terms of letters rather than numbers. This is not in order to be mysterious but to be general. Whereas an equation involving specific numbers refers only to a single situation, an equation involving letters can be very flexible. For example, the equation $5 + 10 = 8 + 7$ is perfectly true, but it would not be true if other numbers were substituted. It refers to a special case. On the other hand, the rule that addition may be done in any order can be expressed by the equation $a + b = b + a$. This equation is always true, no matter what numbers are substituted for a and b. Moreover, this equation indicates clearly that the *same numbers* are used on both sides but used in a

different order. The use of letters, then, permits a clear and general statement to be expressed.

It is very common in scientific work for letters to be used in other ways. A letter is often used as a convenient abbreviation for a longer word or phrase that describes a quantity used in a calculation. For instance, if the mass of aluminum used in a reaction is a quantity that appears repeatedly in a calculation, then it is convenient to write "let a = mass of aluminum used" and to use the letter a rather than the longer phrase thereafter.

Another use of letters is in general equations expressing some relationship. These equations can be used as formulas for calculation. For example, the density of a substance is defined as the mass for every unit of volume. This can be expressed as an equation, where d is the density, m the mass, and v the volume:

$$d = \frac{m}{v}$$

This equation can then be used for future calculations in which one measures any two of the three quantities and then calculates the third.

Although any letter can be used in an equation, letters at the beginning of the alphabet are often used for *constants*, quantities that have a fixed value in the situation under consideration. Letters at the end of the alphabet (x, y, and z) are used to indicate *variables*, quantities that change in a way specified by the mathematical statement. These are common usages rather than firm rules. In addition, in scientific work, certain letters and symbols are used by general agreement as abbreviations for certain measurements (as in the definition of density). These usages will be followed in this book.

Occasionally in science, a two-letter symbol is used. For example, the Greek capital letter delta is used to mean "a change in" so that a change of temperature is shown in an equation as ΔT. The ΔT is all one unit, not a quantity Δ multiplied by a quantity T. Unfortunately, this can be confusing if you are not familiar with the particular notation used. Sometimes the same letter is used for several quantities, with special designations such as subscripts to tell one from another (T_1, T_2, T_f for the first temperature, second temperature, final temperature.)

Let us review some of the common and undoubtedly familiar notations of mathematics and some general rules.

2.1.B. Symbols

Equivalence

Equivalence is shown by the equals sign, " = ." The equals sign is used in mathematics to mean "is another name for," and the two quantities con-

nected by an equals sign can be used interchangeably. The statement $2 + 3 = 5$ means that $2 + 3$ is another name for the number represented by the numeral 5.

For work in chemistry it is convenient to use the same equivalence sign for a broader range of meanings. There is no problem understanding what the equals sign means when it is used to show the same measurement expressed in different units, as

$$\$1 = 4 \text{ quarters}$$

$$1000 \text{ mL} = 1 \text{ L (one thousand milliliters equal one liter)}$$

It is convenient to use an equals sign to connect two quantities that are in some sense equivalent, even where one is not "another name for" the other. For example, if in a reaction two molecules of ammonia, NH_3, react with one molecule of sulfuric acid, H_2SO_4, then in a chemical sense two molecules of NH_3 are equivalent to one molecule of H_2SO_4. So, in a sense, 2 molecules NH_3 = 1 molecule H_2SO_4.

The symbol $\approx$ means "is approximately equal to."

The symbol $\equiv$, which looks like an equals sign with emphasis, is read "is identical to."

It is sometimes useful to indicate that one quantity *is not equal to* another; this is shown by drawing a diagonal line through the equals sign, " $\neq$."

$a \neq b$ is read "a is not equal to b"

Inequalities

Inequalities may be shown with the symbol $>$, *greater than*, or $<$, *less than.*

$a > b$ "a is greater than b"

$b < a$ "b is less than a"

One way to remember which one is which is to note that the large end of the symbol points to the bigger quantity, the small end to the smaller quantity.

A variation on these symbols is used to indicate that one quantity may not be greater than another. A satisfactory relationship exists if the two quantities are equivalent, or if the first is smaller than the second.

$a \leq b$ "a is equal to or less than b"

$b \geq a$ "b is equal to or greater than a"

Plus Sign

The plus sign, " + ," is used in two ways. A plus sign before a number indicates that the number is positive. A plus sign between two numbers indicates that the numbers are to be added.

Minus Sign

A minus sign, " − ," is similarly used both to show that a number is negative and to indicate subtraction. Subtraction is used to find the *difference* between two numbers, that is, the change required to go from one to the other.

Although the same symbols, + and −, are used to show positive and negative numbers and also to indicate the arithmetic operations of addition and subtraction, this does not cause problems. Adding negative 3 to positive 2 is the same as subtracting 3 from 2 and gives the same result.

$$2 + (-3) = 2 - 3 = -1$$

Multiplication

Multiplication is addition repeated a specified numbers of times. Therefore, 3×4, which is read "three times four" or "three fours," means $4 + 4 + 4$. The result is called the *product*. There are several ways to indicate that the operation *multiply* is to be performed. In arithmetic it is common to use an " × " between the *factors* (the numbers to be multiplied). However, in equations in which letters are used, the × might be mistaken for a variable. Therefore, other symbols to indicate multiplication are used. When only simple letters or one number and one or more letters are used, no sign for multiplication is shown, and the letters are simply written in sequence.

ab means a times b

$12x$ means 12 times x

$3ab^2$ means 3 times a times b times b

(the exponent applies only to the b, not to the 3 or a)

Where confusion might arise from writing the quantities in sequence, parentheses are used or, less commonly, a dot or an asterisk is positioned midway between the quantities. The product three times four times x could then be written

$3(4)(x)$ or $3(4x)$ or $3 \cdot 4 \cdot x$

or $3 * 4x$ or even $4(3x)$

The important consideration is to show that the 3 and 4 are to be multiplied and are not meant to be the number 34.

Exponents

Exponents are used to show repeated multiplication. If 2 is to be multiplied by itself and the result again multiplied by 2 (so that 2 is used as a factor 3 times), we can write $2 \times 2 \times 2$ or, more conveniently, 2^3. The 2 is called the *base* and the 3 the *exponent*. The exponent shows how many times the base is used as a factor. Then x^5 shows that x is used as a factor 5 times:

$$x^5 = x \cdot x \cdot x \cdot x \cdot x$$

In even more general terms, x^n means that x is used as a factor n times.

Division

Division can be shown by the division sign, " ÷ ," but is far more frequently shown by writing a fraction. Thus 10 divided by 5 is written

$$\frac{10}{5} \qquad \text{or} \qquad 10/5$$

The horizontal line is clearer and more convenient for use in pencil-and-paper calculations. The diagonal slash is used to indicate division when working on a computer and is usually used in printed material because it permits a single horizontal line of type. The result of division is called the *quotient*.

Division can also be indicated by a negative exponent:

$$x^{-3} = \frac{1}{x^3}$$

In this notation, 10 divided by 5 would be written $10(5^{-1})$. When numbers are written in exponential notation (see Chapter 3), it is convenient to write a negative exponent to indicate division. This notation is being used increasingly for units; for example, miles per hour would be written miles hr^{-1} rather than miles/hr.

Grouping

Grouping is shown by parentheses, which must be used wherever there is any possibility of confusion. Where parentheses are used, the operation grouped within the parentheses is to be performed first. Then the result is used as indicated. For instance, $a(b + c)$ means that the *sum of b and c* is multiplied by a. If the parentheses had been omitted, the statement

$ab + c$ would have meant that only b is multiplied by a, and then c is added to the product. Sometimes parentheses are used only to call attention to a specific grouping. For instance, an important law of arithmetic, the *associative law*, is stated

$$a + (b + c) = (a + b) + c$$

This says that three numbers may be added in any order without changing the results. The parentheses are used to show which two are added first.

Sometimes quantities within parentheses must be grouped further. For this, another set of parentheses inside the first may be used or, to avoid confusion, square brackets or braces (wavy brackets). When braces are used, they are used for the largest (outermost) grouping, followed by square brackets for the next group, and parentheses for the innermost group. An example of this sequence follows:

$$5\left\{[(3 + 9)(1 + 4)] - \frac{cd}{k}\right\}$$

In such an expression, the operations are performed starting with the innermost group and working outward. In the expression shown, the first thing to do is to add $3 + 9$ and add $1 + 4$. Then multiply the results. Then subtract cd/k from the product. Last, multiply the result by 5.

On a calculator or computer, only the simple, curved parentheses are used, not square and wavy brackets. If your calculator can use several levels of parentheses, it is essential to be careful to check that every set of parentheses is closed, that is, that it has both a left and right side.

Roots

Taking a root is the opposite of raising to a power. The fourth root of a number is the factor that must be used four times to produce the number.

$$x = \sqrt[4]{y} \qquad \text{if} \qquad x \cdot x \cdot x \cdot x = y$$

Roots are shown in either of two ways. For square roots, and occasionally for others, it may be convenient to use the sign $\sqrt{\ }$ or $\surd$. The horizontal line is convenient in that it shows that the root is to be taken for everything under the line, but it is sometimes omitted for speed and economy in setting material in type. If a cube or higher root is meant, the numerical index must be shown. The "square root of 25" can be written $\sqrt{25}$, but "the cube root of 27" would be written $\sqrt[3]{27}$. The meaning of "square root of 25" is "the number that when squared (raised to the second power, i.e., multiplied by itself) gives 25." This gives

rise to one problem: Since both $(+5)^2$ and $(-5)^2$ equal 25, which root is meant? When there is no reason to choose one over the other, the correct statement is $\sqrt{25} = \pm 5$, or "the square root of 25 is plus or minus 5."

Another way to indicate roots, and one more common in science, is to use fractional exponents (see Chapter 3). In this notation the exponent 1/2 shows the square root, the exponents 1/4 shows the fourth root, and so on. Then, $(25)^{1/2} = \pm 5$. Both raising to a power and extracting a root can be shown in the same exponent, so $(25)^{3/2}$ means $\sqrt{25 \times 25 \times 25}$. That is, 25 is to be cubed and the square root of the result found. Alternatively, it can be read

$$25^{1/2} \times 25^{1/2} \times 25^{1/2}$$

That is, the square root of 25 is to be cubed. Either sequence of operations gives the same result. (Verify for yourself that the result is ± 125.)

Absolute Value

The absolute value of a quantity is shown by writing the quantity between vertical lines, $|x|$. The absolute value is the positive numerical value, regardless of plus or minus signs. Thus $|+2| = 2$ and $|-2| = 2$.

Equations

Equations are mathematical sentences. An equation shows that one quantity equals (is another name for, or has the same value as) another. It does not matter which of the two quantities is shown on the left and which on the right. In fact, an equation may be switched left for right: that is, if $x = y$, then $y = x$. Furthermore, the two sides remain equivalent even though various mathematical operations are performed. Examples of such operations are

1. Carrying out the operation indicated in the equation.

   $x = 2(4)$ Carry out the multiplication indicated.

   $x = 8$

2. Substituting a number or symbol or quantity for its equivalent.

   (a) $$a + b = c$$

   If $a = 2$ and $b = -7$, then it is possible to substitute 2 for a and -7 for b, giving

   $$2 - 7 = c$$

(b) $b = a(x + y - z^2)$ and $x + y - z^2 = m$

m can be substituted for its equivalent expression in the first equation, giving

$$b = am$$

3. Changing the value of one side of the equation while *making an identical change on the other side.* It is essential to make the same change on both sides: otherwise, the two sides will cease to be equivalent.

(a) $$x - 2 = 8$$

Add 2 to *both* sides:

$$x - 2 + 2 = 8 + 2$$
$$x = 10$$

(b) $$(x - 2)^2 = 8^2$$

Square *both* sides of the equation. It is essential that the entire left side of the equation be squared, not just the x or the 2.

$$x^2 - 4x + 4 = 64$$

2.2. RULES OF MATHEMATICS

Some of the fundamental rules of mathematical operations are given next in symbols and in words.

$a + b = b + a$	Addition may be performed in any order to give the same sum.
$a - a = 0$	The sum of any number and its negative is zero; or, if any number is subtracted from itself, the result is 0.
$a + 0 = a$	Addition of 0 to any number leaves the number unchanged.
$ab = ba$	Multiplication may be performed in any order to give the same product.

$a \cdot 0 = 0$	The product of any number times 0 is 0.
$a \cdot 1 = a$	Multiplication by 1 leaves a number unchanged.
$\frac{a}{a} = 1$ or $a\left(\frac{1}{a}\right) = 1$	Any number (except 0) divided by itself gives a quotient of 1, or the product of a number and its reciprocal is 1.

Many useful rules of mathematical operations are not considered fundamental because they are derived from those just given. However, these rules too are the basis of work with mathematics (including simple arithmetic). Some of the widely used procedures are summarized and examples given in Section 10.1.

2.3. POSITIVE AND NEGATIVE NUMBERS

In algebra, quantities may be either positive or negative. Addition and subtraction that involve negative numbers are often called *algebraic addition* and *algebraic subtraction*. In an equation expressed with letters, a letter might stand for either a positive or a negative quantity. This is quite aside from the plus or minus signs used to show addition and subtraction.

The use of combinations of positive and negative numbers is very common in science, where the opposite signs can be used to show various types of opposites. For instance, motion forward could be considered positive and motion backward negative. Then, moving five steps forward and three back would be represented as a total of $+5 - 3$, which ends up totaling $+2$, or two steps forward from the starting point. "Forward" and "backward" may be purely arbitrary (perhaps "forward" is north and "backward" is south, for instance).

There are a number of situations in science where plus and minus signs are used to designate two opposite effects such that one reverses the other. Again the decision about which is to be called positive and which negative is arbitrary; a definition must be agreed upon so that everyone will use the same convention for the same meaning. For example, it has been accepted by scientists that heat added to a system will be written as positive; heat given off by a system is then written as negative. One of two opposite types of electric charge is called positive, and the opposing charge is called negative.

Let us review the rules for algebraic operations with negative numbers, using the concept of movement in opposite directions to form a mental picture of the results.

2.3.A. Algebraic Addition

Any two numbers may be added algebraically no matter whether they are positive or negative. We can draw a number line as in Figure 2.1, marking 0 in the middle and positive numbers to the right. Addition can then be shown by a series of movements along the line. To add a positive 2, move two spaces to the right; to add a negative 5, move five spaces to the left. These movements may be performed in any order.

EXAMPLE 1

Add the numbers $+2$, -8, and $+5$. Starting at 0, try moving two places to the right, eight left, and five right. Do it in some other sequence, such as eight left, two right, and five right. You should always end up in the place, -1.

$$+2 + (-8) + 5 = 2 - 8 + 5 = -1$$ ■

A common calculation involving positive and negative charges in chemistry is the calculation of the charge on an ion. An ion is defined as a charged particle. It may be monatomic ("one atom," with the nucleus of a single atom) or polyatomic ("many atom," with the nuclei of several atoms joined by chemical bonds), but the number of electrons in an ion is not equal to the number of protons present. Protons and electrons have charges that are equal and opposite. By definition, the charge on the proton is called $+1$ and the charge on the electron is called -1. Therefore, there must be a net charge on an ion because the number of protons (positive charges) does not equal the number of electrons (negative charges).

EXAMPLE 2

What is the net charge on the sodium ion, which has 11 protons, each with a charge of $+1$, and 10 electrons, each with a charge of -1?

$$\begin{array}{lr} \text{Total positive charge: } 11(+1) = & +11 \\ \text{Total negative charge: } 10(-1) = & \underline{-10} \\ \text{Sum} & +1 \end{array}$$ ■

FIGURE 2.1

2.3.B. Algebraic Subtraction

Subtraction is the process of finding the difference between two numbers. On the number line, that means finding how far apart the two numbers lie. Whenever you are asked to calculate the amount of change in something, the operation required is subtraction.

Changes are always calculated by the pattern

$$\text{change} = \text{final position} - \text{initial position}$$

For example, if the temperature was 15°C in the daytime and -12°C at night, you would calculate the drop in temperature by subtracting the first temperature from the second one.

$$-12^\circ\text{C} - (+15^\circ\text{C}) = -27^\circ\text{C}$$

The minus sign shows that the temperature went down. It had to change by 15°C to get down to zero, then another 12°C to get down to -12°C, for a total of 27°C downward. One way to think of this is to draw a picture of a ladder or a staircase. How many steps must you go up or down?

To subtract a number, change the sign of the number and add.

$$+5 - (-3) = +5 + 3 = 8$$
$$+5 - (+3) = +5 - 3 = 2$$
$$-5 - (-3) = -5 + 3 = -2$$

Verify these answers by referring to the number line, Figure 2.1.

To find the difference between $+5$ and -3, locate these two points on the line and count the spaces between them. There are eight places: 3 from -3 to 0 and 5 more from 0 to $+5$. Therefore, the rule for the operation of subtraction must be one that gives a result of 8. Now consider the difference between $+5$ and $+3$. Locate these two points on the number line and count the places between them. Here the difference is 2. Similarly, the difference between -5 and -3 is two places.

2.3.C. Multiplication and Division of Positive and Negative Numbers

Multiplying or dividing two numbers with like signs produces a positive number. If the signs are opposite, the result is negative. In other words,

The product or quotient of two positive numbers is positive.
The product or quotient of two negative numbers is positive.
The product or quotient of a positive number and a negative number is negative.

Think of a minus sign as telling you to turn around and face the opposite way. Multiplication (or division, which can be considered multiplication by a fraction) by a negative number is taken to mean multiply and turn around to face the opposite direction. Multiplication by a second negative number tells you to turn around again, so you are back to facing the original direction.

PROBLEMS

2.1 Add the numbers in each set.

(a) $+3,\ +9,\ -5$ (b) $-6,\ +1,\ +4$
(c) $-5,\ +7,\ -2$ (d) $-15,\ -3,\ -8$
*(e) $+7x,\ -2x,\ -x$ (f) $-5x,\ +x,\ +3x$

2.2 Subtract as shown.

(a) $8-(+3)$ *(b) $8-(-3)$
(c) $3-(+8)$ (d) $3-(-8)$
(e) $5-(-6)$ (f) $6-(-5)$
(g) $2x-(-3x)$ (h) $5a-(+2a)$

2.3 Multiply or divide as indicated.

(a) $2(+8)$ (b) $2(-8)$
(c) $10(-2)$ *(d) $(-1)(-1)$
(e) $-6\div(+3)$ (f) $-3(-5)(-4)$
(g) $x(-2)$ (h) $-5x(-4)$
(i) $\dfrac{3}{-2}$ (divide 3 by -2) (j) $\dfrac{-27}{-9}$
(k) $\dfrac{-6x}{+2}$ (l) $\dfrac{-4x}{-4}$

2.4 Find the net charge on each of the following. Each proton has a charge of $+1$ and each electron has a charge of -1.

	Name	*Number of Protons*	*Number of Electrons*
*(a)	Chloride ion	17	18
(b)	Sulfide ion	16	18
(c)	Calcium ion	20	18
(d)	Argon	18	18
(e)	Nitrate ion	31	32
(f)	Ammonium ion	11	10
(g)	Phosphate ion	47	50
(h)	Aluminum ion	13	10

Ions are chemical particles that have a charge, either positive or negative, usually of 1, 2, or 3 units. Compounds made of ions contain equal numbers of positive and negative charges, so that the compound has no overall charge. That is, the sum of the charges on the positive ions and the charges on the negative ions must be 0. If an ion has a charge of $+2$, it must be associated with one ion having a charge of -2 or two ions each having a charge of -1. If the charges are not even multiples of each other, several ions of one kind must be associated with more than one of the other kind, so that the sum of the charges is still zero. This is necessary since you cannot have a fraction of an atom. By convention, the smallest numbers that will give a sum of zero are used.

PROBLEMS

2.5 For each of the following, the charge is shown as a superscript. The sign is indicated, and the size of the charge is shown if it is not 1. Tell how many ions of each kind must be combined to make a compound with a net charge of zero.
(a) Ca^{2+} and Cl^{-} (The number 1 is not shown.)
(b) Na^{+} and S^{2-}
(c) Al^{3+} and I^{-}
(d) Mg^{2+} and O^{2-}

2.6 Find the numbers of each kind of atom that will give a compound with an overall 0 charge.
*(a) Cr^{3+} and O^{2-} (b) As^{3+} and S^{2-}
(c) Mg^{2+} and N^{3-} (d) P^{5+} and O^{2-}
(e) Al^{3+} and O^{2-}

2.7 Heat added to a system is considered to be a positive quantity. The loss of heat by the system (cooling) is then negative. Calculate the net heat change in each.
*(a) A beaker of water is heated by a flame long enough to add 500 J. Then the flame is removed and the beaker of water cools, losing 200 J. What is the total heat change?
(b) A piece of metal is cooled, losing 375 J. Then it is allowed to warm up, gaining 132 J. What is the total energy change?

2.4. INVERSE OPERATIONS

It is often useful to negate the effect of a mathematical operation. This is done by performing the inverse of the operation. For example, if you earn \$25 (take-home pay) and then pay a bill for \$25, the amount of

money you have at the end is the same as the amount with which you started. You had a \$25 income and a \$25 outgo. One of these changes negates the effect of the other. If one, say income, is considered to be positive, then the other is the opposite, or negative.

$$\$25 - \$25 = 0$$

In general terms, the additive inverse of any number a is $-a$, read "the negative of a."

$$a + (-a) = a - a = 0$$

This is true whether a is positive or negative, since the negative of a negative is a positive.

$$-a - (-a) = -a + a = 0$$

PROBLEM

2.8 What is the additive inverse of each of the following expressions; that is, what must be added to each to make the sum zero?
(a) 2 (b) 79 *(c) -20 (d) $3x$ (e) $27 - 13$
*(f) $x + y$ [*Hint*: If the number $a = (x + y)$, what is $-a$?]
(g) $x - 2$ (h) $x - 7y$ (i) x^2

The inverse operation of multiplication is division, and the inverse of division is multiplication. For either, the operation consists of multiplication by the *reciprocal* or *multiplicative inverse*. This is a number whose product with the original number is 1,

$$a\left(\frac{1}{a}\right) = 1$$

where a and $1/a$ are reciprocals of each other.

Why are we working in terms of 0 for addition and of 1 for multiplication? Think about the effect of using each. Addition of 0 leaves a number unchanged. Therefore, addition of two numbers (a number and its negative) that have a sum of 0 leaves the original unchanged. In multiplication, however, multiplication by 0 gives 0, not the original number. To leave a number unchanged, we must multiply by 1 rather than by 0. Therefore, multiplication by two numbers whose product is 1 leaves a quantity unchanged.

$$x(a)\left(\frac{1}{a}\right) = x \qquad \text{since } a\left(\frac{1}{a}\right) = 1 \quad \text{and} \quad x(1) = x$$

PROBLEM

2.9 What is the multiplicative inverse of each?

(a) m *(b) 4 (c) 25 (d) $3x^2$ (e) 100

(f) 0.01 [***Hint***: What can be multiplied by 0.01 to give 1 as the product?]

(g) 0.5

To find the inverse of a fraction, simply invert the fraction (turn it upside down):

$$\text{the inverse of } \frac{2}{3} \text{ is } \frac{3}{2} \qquad \text{since } \frac{2}{3} \times \frac{3}{2} = 1$$

In effect, we can consider the fraction to be a product:

$$\frac{2}{3} = 2 \times \frac{1}{3}$$

The inverse of 2 is 1/2. The inverse of 1/3 is 3. The inverse of the whole product is the product of the two reciprocals, 3/2.

PROBLEM

2.10 What is the reciprocal of each?

(a) $\frac{1}{4}$ (b) $\frac{2}{8}$ (c) $\frac{9}{7}$ *(d) $\frac{1}{x}$

(e) $\frac{1}{x^2}$ (f) $\frac{3x}{2y}$ (g) $\frac{25}{x^2 + y^2}$

2.5. FRACTIONS

Quantities expressed as fractions are very common in scientific work. The fraction is used to indicate division or to indicate a ratio of one quantity to another. Since such situations are frequently encountered, it is important to be able to handle fractions confidently.

A fraction consists of a *numerator*, the number above the line, and a *denominator*, the number below the line. It indicates multiplication by the numerator and division by the denominator. The fraction 5/9, then, indicates that something is to be multiplied by 5 and divided by 9. These two

operations can, of course, be done in any order; that is, the division by 9 could be carried out before the multiplication by 5 if that is more convenient.

$$\frac{5}{9} \times 36 = \frac{5(36)}{9} \qquad \text{or} \qquad \frac{36}{9}(5) = 20$$

If you recognize that $36/9 = 4$, the second sequence is the simpler one, since the calculations can be done rapidly in your head in two steps: $36/9 = 4$, $4(5) = 20$. The alternative sequence, multiplication by 5 and then division by 9, is more difficult for most people.

A fraction can have a numerator or a denominator of 1. The fraction 1/2 can be taken to show simply division by 2, since multiplication by 1 does not change anything. In general, division by any number a can be written as multiplication by the fraction $1/a$. A whole number can be considered to be a fraction with the denominator 1; division by 1 does not change the number.

A fraction can never have a denominator of 0, since division by 0 is undefined. Any general statement involving fractions should indicate that the statement is true only if the denominator is not 0. However, since this condition is always necessary, it is usually not expressed but is taken to be understood by the reader.

In your early study of arithmetic you probably learned that a fraction like 9/5, in which the numerator is greater than the denominator, is called an *improper fraction* and should be converted to a *mixed fraction*: $9/5 = 1(4/5)$. In scientific work it is far more convenient to leave it as an improper fraction or to convert it to decimal form by dividing the numerator by the denominator: $9/5 = 1.8$.

PROBLEMS

2.11 Express in words the instructions conveyed by each of the following fractions used to multiply some quantity x.

(a) $\frac{2}{3}$ *(b) $\frac{a}{b}$ *(c) $\frac{1}{10}$

(d) $\frac{10}{2}$ (e) $\frac{1}{x}$ (f) $\frac{x}{5}$

2.5.A. Multiplication of Fractions

Fractions are multiplied by multiplying the numerators to produce the new numerator and multiplying the denominators to produce the new denominator.

$$\frac{a}{m} \times \frac{b}{n} = \frac{ab}{mn}$$

$$\frac{1}{2} \times \frac{3}{4} = \frac{1(3)}{2(4)}$$

$$= \frac{3}{8}$$ (Look at this on a ruler and verify that half of 3/4 in. is indeed 3/8 in.)

PROBLEM

2.12 Multiply the fractions shown. Simplify where appropriate.

(a) $\frac{2}{3}\left(\frac{4}{9}\right)$ *(b) $\frac{x}{3}\left(\frac{1}{2}\right)$

(c) $\frac{9}{2}\left(\frac{2}{3}\right)$ *(d) $\frac{3}{x}(2)$

(e) $2\left(\frac{a+b}{y}\right)$ (f) $\frac{1}{2}\left(\frac{1}{a+b}\right)$

(g) $x\left(\frac{1}{x}\right)$ (h) $x\left(\frac{1}{x^3}\right)$

(i) $6.0\ \cancel{\text{cm}} \times \frac{1.2\ \text{g}}{\cancel{\text{cm}}^3}$ (j) $20\ \cancel{\text{g}} \times \frac{1\ \text{cm}^3}{0.80\ \cancel{\text{g}}}$

2.5.B. Division with Fractions

One convenient way to show division by a quantity a is to indicate multiplication by the fraction $1/a$. That is, the divisor becomes the denominator of the fraction. We recognize such operations in speech when we say "divide in half" and mean divide by 2. Once this notation is used, the calculation is simply multiplication by a fraction. For example, 2/10 divided by 3 is expressed as 2/10 times 1/3:

$$\frac{2}{10} \times \frac{1}{3} = \frac{2}{30}$$

To divide a/b by c, write

$$\frac{a}{b} \div c = \frac{a}{b}\left(\frac{1}{c}\right) = \frac{a}{bc}$$

It is sometimes possible to simplify even before the fractions are multiplied, since multiplication can be done in any order.

$$\frac{2}{5} \div 2 = \frac{2}{5} \times \frac{1}{2} = \frac{2}{2} \times \frac{1}{5} = 1 \times \frac{1}{5} = \frac{1}{5}$$

PROBLEM

2.13 Divide each of the fractions by the quantity shown.

*(a) $\frac{1}{2} \div 2$ (b) $\frac{2}{10} \div 2$

(c) $\frac{3}{10} \div 5$ (d) $\frac{x}{y} \div a$

*(e) $\frac{x}{y} \div x$ (f) $\frac{x}{y} \div y$

(g) $\frac{2x}{y} \div 2y$ (h) $\frac{a}{b} \div (x + y)$

(i) $\frac{1}{a} \div (a + b)$

The same procedure is followed for division by a fraction, but it may look a bit different. Again division is accomplished by multiplying by a fraction that has the divisor as denominator. This amounts to multiplying by the inverse of the divisor. **To divide by a fraction, invert the fraction, and multiply.**

The invert-and-multiply rule is a short way of describing a procedure for removing the fraction from the denominator. "Removing the fraction" means converting the denominator to 1. This can be accomplished by multiplying the denominator by its inverse. However, if the denominator is multiplied, the numerator must be multiplied by the same thing to keep the value unchanged. Therefore, as the denominator is converted to a value of 1, the numerator must be multiplied by the inverse of the original denominator.

EXAMPLE 3

Divide 5 by 2/3.

Following the invert-and-multiply procedure, we obtain

$$5 \div \frac{2}{3} = 5 \times \frac{3}{2} = \frac{15}{2}$$

Alternatively, we could write

$$5 \div \frac{2}{3} = \frac{5}{\frac{2}{3}}$$

Multiply both numerator and denominator by the multiplicative inverse of the denominator. (In other words, invert the denominator and use this to multiply both numerator and denominator.)

$$\frac{5}{\frac{2}{3}} \times \frac{\frac{3}{2}}{\frac{3}{2}} = \frac{5\left(\frac{3}{2}\right)}{1} = \frac{15}{2} \qquad \blacksquare$$

■ EXAMPLE 4

Divide $\frac{a}{b}$ by $\frac{x}{y}$.

$$\frac{a}{b} \div \frac{x}{y} = \frac{\frac{a}{b}}{\frac{x}{y}} = \frac{a}{b} \times \frac{y}{x} = \frac{ay}{bx} \qquad \blacksquare$$

■ EXAMPLE 5

Divide 4 by $1/(x + y)$.

$$4 \div \frac{1}{x + y} = \frac{4}{\frac{1}{x + y}} = 4\left(\frac{x + y}{1}\right) = 4(x + y) \qquad \blacksquare$$

■ EXAMPLE 6

Divide $[H^+][OH^-]$ by $\dfrac{[H^+][A^-]}{[HA]}$.

Each set of square brackets with its enclosed symbol is a single factor. (The meaning of the notation is "the concentration of the species shown within the brackets.") In other words, the problem above is essentially

$ab \div (ac/d)$, where $a = [H^+]$, $b = [OH^-]$, $c = [A^-]$, and $d = [HA]$.

$$\frac{[H^+][OH^-]}{\dfrac{[H^+][A^-]}{[HA]}} = \frac{[H^+][OH^-] \times [HA]}{[H^+][A^-]}$$

$$= \frac{[OH^-][HA]}{[A^-]}$$

Notice that the term $[H^+]$ appears in both the numerator and denominator. When a quantity is divided by itself, the result has the value of 1, which need not be written. ■

PROBLEMS

2.14 Divide as indicated and, where possible, simplify the answer.

*(a) $2 \div \frac{1}{2}$ (b) $2 \div \frac{1}{10}$ (c) $10 \div \frac{5}{2}$

(d) $a \div \frac{b}{c}$ (e) $5a \div \frac{2a}{3}$ (f) $\frac{\text{g}}{\text{mL}} \div \text{mL}$

(g) $\text{g} \div \frac{\text{g}}{\text{mole}}$

2.15 Divide as indicated and simplify the results where possible.

(a) $\frac{1}{2} \div \frac{1}{2}$ *(b) $\frac{1}{2} \div \frac{1}{10}$

(c) $\frac{1}{a} \div \frac{2}{a}$ (d) $\frac{x}{y} \div \frac{y}{x}$

*(e) $\frac{x}{y} \div \frac{1}{a+b}$ (f) $\frac{x}{y} \div \frac{x}{x+y}$

(g) $\frac{\text{g}}{\text{mole}} \div \frac{\text{L}}{\text{mole}}$

2.16 Divide $[Ag^+][NO_2^-]$ by $\dfrac{[H^+][NO_2^-]}{[HNO_2]}$.

***2.17** Divide $\dfrac{[H^+][A^-]}{[HA]}$ by $[H^+][OH^-]$.

2.5.C. Canceling

Fractions can be simplified by dividing both the numerator and the denominator by the same quantity. Since this is equivalent to multiplying by a fraction that has a value of 1, it does not change the size of the original fraction.

A useful notation for this process is canceling, that is, drawing a line through the factors that have been used. This helps you keep track of what you have done. The process, then, is to identify any factors that appear both in the numerator and in the denominator and cancel these factors.

$$\frac{6}{8} = \frac{2 \times 3}{2 \times 4}$$

$$\frac{2 \times 3}{2 \times 4} = \frac{2}{2} \times \frac{3}{4}$$ since multiplication can be done in any order

$$\frac{2}{2} \times \frac{3}{4} = 1\left(\frac{3}{4}\right) = \frac{3}{4}$$ since any number divided by itself has a value of 1 and multiplication by 1 does not change a number

More commonly, the intermediate steps are not written and the process is shown in one of two ways:

$$\frac{6}{8} = \frac{\cancel{2} \times 3}{\cancel{2} \times 4} = \frac{3}{4}$$

or

$$\frac{\overset{3}{\cancel{6}}}{\underset{4}{\cancel{8}}} = \frac{3}{4}$$

In the latter method the factor 2 is implied but not written. **Only factors used in multiplication can be canceled. Terms used in addition or subtraction cannot be canceled.**

EXAMPLE 7

Simplify by canceling.

(a) $\dfrac{3a}{10a} = \dfrac{3\cancel{a}}{10\cancel{a}} = \dfrac{3}{10}$

(b) $\dfrac{3a^2}{10a} = \dfrac{3a \cdot \cancel{a}}{10\cancel{a}} = \dfrac{3a}{10}$

(c) $\dfrac{6x + 4y}{2} = \dfrac{\cancel{2}(3x + 2y)}{\cancel{2}} = 3x + 2y$

Units such as ft or cm can be treated as if they are factors and can be multiplied or divided in the same way. In doing so, ignore any difference between singular and plural words; "feet" and "foot" are the same unit.

EXAMPLE 8

Perform the operations indicated and simplify.

(a) $3\text{ ft}(2\text{ ft}) = 6\text{ ft}^2$

(b) $\dfrac{3\text{ cm} \times 2\cancel{\text{ cm}} \times 6\cancel{\text{ cm}}}{1\cancel{\text{ cm}} \times 4\cancel{\text{ cm}}} = \dfrac{36\text{ cm}}{4} = 9\text{ cm}$

(c) $\dfrac{7\text{ A} \times \cancel{5}\text{ sec} \times 1\cancel{\text{ min}}}{\cancel{5}\cancel{\text{ min}}} = 7\text{ A-sec}$ ■

PROBLEMS

2.18 Simplify each fraction that can be simplified. Identify those that cannot be simplified by canceling.

(a) $\dfrac{5}{10}$ *(b) $\dfrac{25}{5}$

(c) $\dfrac{36}{9}$ (d) $\dfrac{36x}{5x}$

(e) $\dfrac{36}{4+9}$ (f) $\dfrac{ab}{ac}$

*(g) $\dfrac{a^2}{ac}$ (h) $\dfrac{a^2b}{ac}$

(i) $\dfrac{x^2y^2}{xyz}$ *(j) $\dfrac{x^2+xy}{x}$

(k) $\dfrac{x}{x^2-xy}$ (l) $\dfrac{x}{x+y}$

(m) $\dfrac{(a+b)(x+y)}{x+y}$

2.19 Simplify each fraction. Treat the units as factors also.

(a) $\dfrac{6\text{ ft}}{3\text{ ft}}$ (b) $\dfrac{9\text{ ft}^2}{3\text{ ft}}$ (c) $\dfrac{2\text{ cm} \times 7\text{ cm} \times 5\text{ cm}}{10\text{ cm}}$

(d) $\dfrac{47\text{ m}^3}{47\text{ m}}$ (e) $\dfrac{75\text{ cm}^2}{5.0\text{ cm}}$

2.5.D. Separating a Fraction Into Two or More Fractions

When the numerator of a fraction is a sum or difference of several terms, the fraction can be expressed as the sum or difference of the several fractions, each with the *same denominator as the original fraction* but with one of the several terms as a numerator.

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

Further simplification of each resulting fraction is sometimes possible and desirable.

EXAMPLE 9

Separate each into the sum or difference of two fractions and simplify.

(a) $\dfrac{x-y}{y} = \dfrac{x}{y} - \dfrac{y}{y} = \dfrac{x}{y} - 1$

(b) $\dfrac{x+y}{xy} = \dfrac{x}{xy} + \dfrac{y}{xy} = \dfrac{1}{y} + \dfrac{1}{x}$ ■

Note that the *entire denominator* is used for each fraction (although later simplification may occur). Even if the denominator is the sum or difference of several quantities, the entire denominator must be used.

PROBLEM

2.20 Express as the sum or difference of two fractions. Then simplify where possible.

*(a) $\dfrac{x+y}{x}$ (b) $\dfrac{9+x}{3}$ (c) $\dfrac{9-x}{x}$

(d) $\dfrac{9+x^2}{x}$ (e) $\dfrac{\pi r^2 + r^2 h}{r^3}$

2.6. VALUES OF FRACTIONS

It is useful to think about the size of a quantity expressed as a fraction or ratio. You can use this information to help you spot wrong answers when you do calculations with fractions.

If a number is multiplied by 1, the value does not change. Similarly, if it is multiplied by a fraction having a value of 1, the value of the original quantity does not change. Since any number divided by itself gives a quotient of 1, a fraction that has the numerator and denominator equivalent to each other has a value of 1.

$$\frac{4}{4} = 1$$

$$\frac{2(2)}{4} = 1 \qquad \frac{3(4)}{2(6)} = 1$$

(Note that the numerator and the denominator need not be written in the same manner as long as they are equivalent.)

$$\frac{1 \text{ foot}}{12 \text{ inches}} = 1 \quad \text{since } 1 \text{ foot} = 12 \text{ inches}$$

$$\frac{1000 \text{ mL}}{1 \text{ liter}} = 1 \quad \text{since } 1000 \text{ mL} = 1 \text{ liter}$$

(These are useful as conversion factors. See Chapter 5.)

If the numerator of a fraction is greater than the denominator, the value of the fraction is greater than 1. Multiplication of any number a by such a fraction gives a product that is larger than a:

$$20 \times \frac{4}{2} = 40 \qquad \frac{4}{2} > 1 \qquad 40 > 20$$

If the numerator of a fraction is smaller than the denominator, the fraction has a value less than 1. Multiplication of a number a by such a fraction gives a product that is smaller than a.

$$20 \times \frac{2}{4} = 10 \qquad \frac{2}{4} < 1 \qquad 10 < 20$$

One way to remember these rules is that a fraction with a bigger top makes the value go up. A fraction with a bigger bottom makes the value go down.

PROBLEM

2.21 Tell whether the value of each fraction is 1, greater than 1, or less than 1.

*(a) $\frac{2}{4}$	(b) $\frac{4}{4}$	(c) $\frac{6}{4}$
(d) $\frac{x}{x}$	*(e) $\frac{2x}{x}$	(f) $\frac{1000}{10}$
(g) $\frac{27}{92}$	(h) $\frac{0.25\text{ ft}}{3\text{ in.}}$	*(i) $\frac{100\text{ mL}}{1\text{ L}}$
(j) $\frac{1\text{ L}}{100\text{ mL}}$	(k) $\frac{10\text{ mm}}{1\text{ cm}}$	(l) $\frac{10\text{ cm}}{100\text{ mm}}$

2.7. RATIOS

A ratio is the relationship between two or more quantities. For example, all the houses in one block of a tract had their landscaping done by the same company, which planted one tree and four bushes in front of each house. On that street there was a ratio of one tree to every four bushes to every one house. The word "every" is usually omitted from such statements and the symbol ":" can be used for "to." We would then write: "The ratio of trees to bushes to houses is 1 : 4 : 1." Sometimes the ratio of two quantities is shown by writing one above the other like a fraction. (This is not suitable, of course, for more than two.) Then the ratio of trees to bushes is 1/4 and the ratio of trees to houses is 1/1, read "one to four" and "one to one." It is essential to make clear which quantity is which; the first one named always goes on top. This notation makes good sense mathematically, since the operation required to find the ratio in simplest terms is one of division, and the fraction implies division. Furthermore, as with fractions, multiplying and dividing each term of the ratio by the same quantity does not change the proportions.

If our gardener had bought 16 arborvitae bushes for the 8 houses on the block, how many arborvitae bushes are there for each house; that is, what is the ratio of arborvitae bushes to houses? It is easy to see that there are 2 bushes for every 1 house. The mathematical operation required is division, which is usually written as a fraction:

$$\frac{16\text{ bushes}}{8\text{ houses}} = \frac{2\text{ bushes}}{1\text{ house}}$$

In this fraction, read "two bushes to every house" or "two bushes per house."

It is often useful to work with two ratios that are equal. We can say that the ratio of a to b is the same as the ratio of c to d, or, in other

terms, the *proportion* of a to b is the same as the proportion of c to d. This can be written

$$a : b = c : d$$

or

$$\frac{a}{b} = \frac{c}{d}$$

Both are read "a is to b as c is to d." For example, the proportion of 8 to 4 is the same as the proportion of 2 to 1, or the ratio of 8 to 4 is the same as the ratio of 2 to 1. When the ratio is written as a fraction, this looks like the old procedure of simplifying fractions:

$$\frac{8}{4} = \frac{2}{1}$$

EXAMPLE 10

It is found that a sample of a compound is made up of 10 sodium atoms (Na), 5 carbon atoms (C), and 15 oxygen atoms (O). What is the ratio in simplest terms (smallest whole numbers) between the numbers of atoms of the various kinds?

The ratio is 10 Na : 5 C : 15 O. Just as with fractions, a ratio may be multiplied or divided without changing its value (i.e., the resulting ratio will be in the same proportions as the original ratio) as long as the same thing is done to every term. Here all the numbers are divisible by 5.

$$10 \text{ Na} : 5 \text{ C} : 15 \text{ O} = 2 \text{ Na} : 1 \text{ C} : 3 \text{ O}$$

It may help to draw a picture. Group the atoms in such a way that each group has the same number of atoms of each type (Figure 2.2). ■

In chemistry this ratio of atoms in a compound would be written Na_2CO_3, where the subscript after a symbol tells the number of atoms

Na Na	Na Na	Na Na	Na Na	Na Na
C	C	C	C	C
O O O	O O O	O O O	O O O	O O O

FIGURE 2.2

of that type, the subscript 1 being omitted. The formula then states that the compound contains 2 atoms of sodium for every 1 atom of carbon for every 3 atoms of oxygen.

EXAMPLE 11

In a chemical reaction two molecules of NH_3 (ammonia) are known to react with every molecule of H_2SO_4 (sulfuric acid). If this ratio is always true, how many molecules of H_2SO_4 react with 28 molecules of NH_3?

The proportion of NH_3 to H_2SO_4 is always 2 : 1.

$$\frac{1 \text{ molecule } H_2SO_4}{2 \text{ molecules } NH_3} = \frac{x \text{ molecules } H_2SO_4 \text{ needed}}{28 \text{ molecules } NH_3}$$

You may be able to see already that x must be 14. The equation can be solved for the number of molecules of H_2SO_4 by multiplying both sides of the equation by "28 molecules of NH_3" (see Chapter 7).

$$\text{molecules } H_2SO_4 \text{ needed} = \frac{1 \text{ molecule } H_2SO_4}{2 \text{ molecules } NH_3} \times 28 \text{ molecules } NH_3$$

$$= 4 \text{ molecules } H_2SO_4$$

In the last equation the units work out right. That is, the unit "molecules of NH_3" appears in both the numerator and the denominator and, therefore, cancels out, leaving only the unit we want, "molecules of H_2SO_4." ■

PROBLEMS

2.22 What is the ratio of the first object named to the second?

(a) 20 children among 8 families
(b) 400 college students to 22 faculty members
(c) 50 boys and 40 girls
(d) 20 nails for 5 shelves

2.23 Give the ratio of atoms in each compound. Write the ratio as a formula. (Use the numbers as subscripts.)

(a) 20 hydrogen atoms (H) and 10 oxygen atoms (O).
(b) 20 carbon atoms (C), 60 hydrogen atoms, and 10 oxygen atoms (O).
(c) 20 sodium atoms (Na), 50 chromium atoms (Cr), 350 oxygen atoms (O). (Express as a ratio of whole numbers.)
(d) 16 iron atoms (Fe) and 24 oxygen atoms (O).

2.8. PROPORTIONS AND EQUATIONS FOR PROPORTIONALITY

The concept of proportionality is one that is very commonly encountered in everyday life as well as in scientific work. The cost of fruit is proportional to the number of pounds or to the number of pieces of fruit. The cost of lumber or cloth or wrapping paper is proportional to the length purchased. The cost of a telephone call is proportional to the length of time of the call.

Similar ideas apply in scientific work. The amount of heat given off by burning a substance is proportional to the amount of the substance burned. The volume of a liquid is proportional to the mass of the liquid.

Each of these statements can be converted to an equation that can be used for calculations. The proportionality sign is replaced by an equals sign and a *proportionality constant*. The proportionality constant tells what number must be used to make the equation true.

$$\text{If} \quad x \propto y$$

$$\text{then} \quad x = ky$$

In the everyday examples given, the proportionality constant is the price for one item, the dollars per item.

EXAMPLE 12

Some paper comes in rolls and is sold at a price of 60 cents a foot. Express this as a proportion and write an equation for calculating the cost of a given length of paper.

$$\text{cost} \propto \text{length}$$

$$\text{dollars} = \frac{\$0.60}{\text{foot of paper}} \times \text{number of feet of paper}$$

Notice that the actual units must match the units of the proportionality constant. If the cost had been given as 60¢, the answer would have been in cents, not dollars. If the cost had been given as price per inch, the length would have had to be expressed in inches. ■

In scientific work, the numerical value of the proportionality constant is determined by experiment, that is, by measuring the sizes of the quan-

tities x and y and calculating what value of the constant makes the equation true. Some proportionality constants appear in so many equations that they are given special names. For example, there is a gas constant symbolized by the letter R that appears in many equations describing the behavior of molecules. Planck's constant, symbolized by the letter h, appears in equations describing the behavior of electrons and in equations describing electromagnetic radiation.

In the preceding relationship, an increase in one quantity produces a proportional increase in the other quantity. This is called *direct proportionality*. It is also possible to have *inverse proportionality*, in which an increase in one quantity produces a decrease in the other.

$$y \propto \frac{1}{x} \qquad \text{or} \qquad y = \frac{k}{x}$$

For example, if you apply pressure to a sample of a gas, the gas can be compressed; that is, the gas occupies a proportionally smaller volume. This proportionality can be expressed as

$$\mathrm{P} \propto \frac{1}{\mathrm{V}} \qquad \text{or} \qquad \mathrm{P} = \frac{\mathrm{R}n\mathrm{T}}{\mathrm{V}}$$

where R is the gas constant, n is the amount of gas, and T is the temperature. For a given sample of gas at constant temperature, n and T are constant; that is, they do no change during the process.

Various types of proportionality are discussed further in Section 7.4.

PROBLEM

2.24 Write each as an equation.

(a) The cost of beverages for the party is proportional to the number of cans purchased. Each can costs $1.25.

(b) The amount of heat needed to heat a gram of water is proportional to the temperature change. You need 4.18 J for every degree Celsius the water is heated.

(c) The amount of heat given off by burning "isooctane" (the octane of gasoline) is proportional to the mass of gasoline burned. 44.7 kJ of heat is given off for every gram of isooctane burned.

2.9. PERCENTS

The term *percent* means "of every hundred." The statement "50% of the class got scores above 60" means that the ratio of the number of students earning scores above 60 to the number of students in the entire class was the same as the ratio of 50 to 100. Expressed as a proportion,

$$\frac{\text{number of students with scores above 60}}{\text{total number of students in the class}} = \frac{50}{100} = 50\%$$

There is no implication that there were actually 100 students in the class. Whatever total number was present, the percent tells what fraction of that total number falls into the particular category:

$$\%x = \frac{\text{number or quantity having the characteristic } x}{\text{total number or quantity present}} \times 100\%$$

The numerator and denominator must be expressed in the same units.

Fractions can easily be converted into percents. This procedure is so common that people tend to use the terms "50%" and "half" interchangeably. There are two ways to think of the procedure for converting a fraction into a percent. One is to remember that "the whole thing" is 100%; a given fraction of the whole is that fraction of 100%. Then 1/4 is 1/4 of 100%:

$$\frac{1}{4} \times 100\% = 25\%$$

Notice that the word "of" in the statement becomes multiplication when the statement is expressed as an equation.

The other way to convert a fraction to a percent is to convert the fraction to a decimal, by dividing the numerator by the denominator. Then move the decimal point two places to the right and add the percent sign. This has the effect of multiplying the decimal by 100% so that this second procedure is really the same as the first, with the steps performed in a different order.

To express a percent as a decimal or fraction, write the percent as a fraction of 100. Thus 6% is 6 out of 100, which can be written 6/100, or 0.06. You can think of the procedure for converting a percent into its decimal equivalent as moving the decimal point two places to the left

and dropping the percent sign. Some calculators have a percent key; its function is to multiply and move the decimal point two places.

■ EXAMPLE 13

A state charges 6% sales tax on purchases. What is the tax on a purchase of $15.20?

Six percent of the price is 6/100 of the total price, or 0.06 times the price.

$$\frac{6}{100}(\$15.20) = 0.06(\$15.20) = \$0.91 \text{ tax} \qquad \blacksquare$$

■ EXAMPLE 14

A 2.00-g sample of a mixture contains 0.420 g of chloride. What is the percent chloride in the mixture?

$$\begin{aligned} \%\text{ chloride} &= \frac{\text{grams of chloride}}{\text{grams of mixture}} \times 100\% \\ &= \frac{0.420 \text{ g chloride}}{2.00 \text{ g mixture}} \times 100\% \\ &= 21.0\% \qquad \blacksquare \end{aligned}$$

■ EXAMPLE 15

A sample contains 27.3% carbon. How many grams of carbon are present in 0.495 g of the sample?

There are several ways to solve the problem. One way is to express the percent as a fraction. The mass of carbon present is this fraction of the total mass of the sample.

$$\begin{aligned} \text{g carbon} &= \frac{27.3}{100} \times \text{g sample} = 0.273 \times \text{g sample} \\ &= \frac{27.3}{100} \times 0.495 \text{ g} = 0.273 \times 0.495 \text{ g} \\ &= 0.135 \text{ g} \end{aligned}$$

Another method is to substitute the data into the equation for percent, as in Example 14 and solve for "g carbon." (See Chapter 7.)

$$\% \text{ carbon} = \frac{\text{grams of carbon}}{\text{grams of sample}} \times 100\%$$

$$27.3\% \text{ carbon} = \frac{\text{g carbon}}{0.495 \text{ g sample}} \times 100\%$$

$$\text{g carbon} = 0.135 \text{ g}$$

The problem can also be solved by using ratios. The term 27.3% means that the ratio of the mass of carbon to the mass of the entire sample is 27.3 : 100. This ratio will be true whatever quantity of sample is used. Therefore, the problem can be treated as one of equal proportions:

$$\frac{27.3 \text{ g carbon}}{100 \text{ g sample}} = \frac{x \text{ g carbon}}{0.495 \text{ g sample}}$$

$$x = \frac{27.3 \text{ g carbon}}{100 \text{ g sample}}(0.495 \text{ g sample}) \quad \text{multiplying both sides by "0.495 g sample"}$$

$$= 0.135 \text{ g}$$

EXAMPLE 16

Commercial hydrochloric acid is sold as a 37% solution of HCl in water; that is, 37% of the mass of the solution is HCl. What mass of solution would be needed to obtain 4.8 g of HCl?

$$\% \text{ HCl} = \frac{\text{g HCl}}{\text{g solution}} \times 100\%$$

$$37\% \text{ HCl} = \frac{4.8 \text{ g HCl}}{\text{g solution}} \times 100\%$$

$$\text{g solution} = \frac{4.8 \text{ g HCl}}{37\% \text{ HCl}} \times 100\%$$

$$= 12 \text{ g}$$

2.10. PERCENT ERROR OR PERCENT DEVIATION

It is often useful to indicate the extent to which an experimentally measured value of a quantity differs from the true value, or to indicate how much variation there is in a group of experimental measurements.

If the true value of a quantity is known, it is possible to calculate the percent error. The error is defined as the difference between the correct value and that measured in an experiment. The absolute value of the difference is used; that is, the numerical value is used without any indication of whether it is positive or negative.

$$\% \text{ error} = \frac{|\text{correct value} - \text{experimental value}|}{\text{correct value}} \times 100\%$$

$$= \frac{|\text{error}|}{\text{correct value}} \times 100\%$$

EXAMPLE 17

A student measured the molecular weight of a compound as 184 g/mole. The correct molecular weight was 182 g/mole. What was the percent error in the experiment?

$$\% \text{ error} = \frac{|\text{true value} - \text{experimental value}|}{\text{true value}} \times 100\%$$

$$= \frac{|182 \text{ g/mole} - 184 \text{ g/mole}|}{182 \text{ g/mole}} \times 100\%$$

$$= \frac{2 \text{ g/mole}}{182 \text{ g/mole}} \times 100\%$$

$$= 1\%$$

Notice that the percent error is a positive number, since the absolute value of the error was used. ■

Often, the correct value of a quantity is not known; the purpose of an experiment is to measure the value. Then the average of several experimental values is taken as the correct value. The difference between this average value and each individual measured value is usually called the *deviation*. A percent deviation can be calculated just like the percent error. Often, rather than a percent deviation for each individual measurement, an average of the several deviations is calculated: This is called the *average deviation*. In reporting the results of an experiment, scientists report the average deviation, or calculate the percent average deviation or a related unit such as "parts per thousand," in order to give information about the precision of the measurement.

EXAMPLE 18

The heat produced in a reaction was measured. In three successive runs on equal samples, the values obtained were 1.39 J, 1.45 J, and 1.27 J.

Calculate the average value for the heat, the average deviation, and the percent average deviation.

The average value is calculated by adding the three experimental values and dividing by the number of experiments.

$$\begin{array}{r} 1.39\ \text{J} \\ 1.45\ \text{J} \\ \underline{1.27\ \text{J}} \\ 4.11\ \text{J} \end{array}$$

$$\frac{4.11\ \text{J}}{3} = 1.37\ \text{J}$$ This is the average value.

To find the deviations, calculate the absolute value of the difference between each experimental value and the calculated average. The average deviation is the average of these differences.

$$|1.37 - 1.39| = 0.02\ \text{J}$$
$$|1.37 - 1.45| = 0.08\ \text{J}$$
$$|1.37 - 1.27| = \underline{0.10\ \text{J}}$$
$$0.20\ \text{J}$$

$$\frac{0.20\ \text{J}}{3} = 0.07\ \text{J}$$ This is the average deviation.

To find the percent average deviation, divide by the average value and multiply the result by 100%.

$$\frac{\text{average deviation}}{\text{average value}} \times 100\% = \frac{0.07\ \text{J}}{1.37\ \text{J}} \times 100\% = 5\%$$ ■

There are several other common ways of expressing the error. One is to express it as "parts per thousand," abbreviated ppt, that is, as a fraction of 1000 instead of as a fraction of 100%. The error in Example 18 would be,

$$\frac{0.07\ \text{J}}{1.37\ \text{J}} \times 1000 = 50\ \text{ppt}$$

The experimental value can also be reported as the average value plus or minus the average deviation, 1.37 ± 0.07 J.

PROBLEMS

2.25 In a football game, a quarterback threw 20 passes, of which 12 were completed. What percent of the passes were completed?

2.26 What is the percent iron in each compound?
*(a) 3.00 g of compound A contains 2.33 g of iron.
(b) 2.50 g of compound B contains 1.59 g of iron.
(c) 0.920 g of compound C contains 0.644 g of iron.
(d) 5.73 g of compound D contains 2.12 g of iron.

2.27 What is the percent sulfate in a mixture if 0.597 g of the mixture is found to contain 0.213 g of sulfate?

2.28 A compound contains 46.0% oxygen. How many grams of oxygen are present in each of the following samples of the compound?
(a) 100 g (b) 200 g *(c) 150 g
(d) 20.0 g (e) 0.592 g

2.29 A mixture contains 95.0% H_2SO_4 (sulfuric acid). How many grams of the mixture are needed to obtain the specified quantities of pure H_2SO_4?
*(a) 5.60 g (b) 100 g (c) 0.120 g

2.30 A student determined that a sample contained 30.2% sulfate. If the sample actually contained 31.5% sulfate, what was the percent error in the student's result?

2.31 A rapid mental calculation performed on a problem gave an answer of 120 mL. When the calculation was performed in detail, the correct answer was found to be 122 mL. What percent error resulted from the rapid calculation?

2.32 A student attempted to determine the concentration of a solution, with no more than 1% average deviation in the results. If the results obtained were 0.5923 *M*, 0.5917 *M*, and 0.5954 *M*, what was the percent average deviation? Was it less than 1%?

SOLUTIONS TO STARRED PROBLEMS

2.1(e) For quantities like $7x$, add the coefficients; if no coefficient is shown, the coefficient is 1. Since $+7 - 2 - 1 = +4$,

$$+7x - 2x - x = +4x$$

2.2(b) To subtract a negative number, change the sign and add.

$$8 - (-3) = 8 + (+3) = 8 + 3 = 11$$

Check this on the number line and verify that there are 11 places between the $+8$ and the -3.

2.3(d) The product of two negative numbers is positive. Therefore,

$$(-1)(-1) = +1$$

2.4(a)
$$\begin{array}{lcr} \text{17 protons} & = 17(+1) = & +17 \\ \text{18 electrons} & = 18(-1) = & \underline{-18} \\ \quad \text{net charge is the sum} & & -1 \end{array}$$

2.6(a) The total charge must be 6, the lowest common multiple of the charges on the two ions. A charge of $+6$ requires two Cr^{3+} ions, each with a charge of $+3$. A charge of -6 requires three O^{2-} ions, each with a charge of -2.

$$2(+3) + 3(-2) = +6 - 6 = 0$$

The compound is therefore Cr_2O_3.

2.7(a)
$$\begin{array}{lcr} \text{Heat added} & = & +500\ \text{J} \\ \text{Heat lost} & = & \underline{-200\ \text{J}} \\ \quad \text{Total} & & +300\ \text{J} \end{array}$$

2.8(c) $-(-20) = +20$. Verify

$$-20 + (+20) = 0$$

(f) $-(x + y) = -x - y$. Verify

$$x + y - x - y = 0.$$

2.9(b) The inverse of a number a is $1/a$. Therefore, the inverse of 4 is $1/4$. To check, $4(1/4) = 1$.

2.10(d) Inverting the fraction $1/x$ gives $x/1$, usually written x. Check

$$\frac{1}{x}(x) = 1$$

2.11(b) The numerator shows multiplication and the denominator shows division. Therefore, the fraction a/b shows that x is to be multiplied by a and divided by b.

(c) Multiplication by 1 does not change a quantity. Therefore, the fraction 1/10 can be read simply as division of x by 10.

2.12(b) To multiply the fraction, multiply the numerators to find the new numerator and multiply the denominators to find the new denominator.

$$\frac{x}{3}\left(\frac{1}{2}\right) = \frac{x(1)}{3(2)} = \frac{x}{6}$$

(d) Consider the whole number 2 as a fraction with a denominator 1.

$$\frac{3}{x}(2) = \frac{3}{x}\left(\frac{2}{1}\right) = \frac{6}{x}$$

2.13(a) Express division as multiplication by the reciprocal.

$$\frac{1}{2} \div 2 = \frac{1}{2}\left(\frac{1}{2}\right) = \frac{1}{4}$$

(e) Express division as multiplication by the reciprocal

$$\frac{x}{y} \div x = \frac{x}{y}\left(\frac{1}{x}\right) = \frac{\cancel{x}}{\cancel{x}y} = \frac{1}{y}$$

The x in the numerator could have been canceled with the x in the denominator before multiplying; two steps are shown here to make the procedure clear.

2.14(a) To divide by a fraction, invert the fraction and multiply.

$$\frac{2}{\frac{1}{2}} = 2\left(\frac{2}{1}\right) = 4$$

2.15(b) To divide by a fraction, invert the fraction and multiply.

$$\frac{\frac{1}{2}}{\frac{1}{10}} = \frac{1}{2}\left(\frac{10}{1}\right) = \frac{10}{2} = 5$$

(e) The $(a + b)$ is treated as a single term.

$$\frac{\frac{x}{y}}{\frac{1}{a+b}} = \frac{x}{y}\left(\frac{a+b}{1}\right) = \frac{x(a+b)}{y}$$

2.17

$$\frac{\frac{[H^+][A^-]}{[HA]}}{[H^+][OH^-]} = \frac{[H^+][A^-]}{[HA][H^+][OH^-]} = \frac{[A^-]}{[HA][OH^-]}$$

2.18(b) The factor 5 is common to both numerator and denominator.

$$\frac{25}{5} = \frac{\cancel{5}(5)}{\cancel{5}} = \frac{5}{1} = 5$$

(g) The factor a is common to both numerator and denominator. Rewrite the a^2 as $a(a)$; the exponent 2 tells you that the a is used as a factor two times.

$$\frac{a^2}{ac} = \frac{\cancel{a}(a)}{\cancel{a}c} = \frac{a}{c}$$

(j) Since there is an x in every term of the numerator, it can be factored out (see Table 10.1).

$$\frac{x^2 + xy}{x} = \frac{\cancel{x}(x + y)}{\cancel{x}} = x + y$$

2.20(a) Each fraction must have the denominator of the original.

$$\frac{x + y}{x} = \frac{x}{x} + \frac{y}{x} = 1 + \frac{y}{x}$$

2.21(a) A fraction has a value of less than 1 if the numerator is smaller than the denominator. Here the numerator, 2, is smaller than the denominator, 4, so the value of the fraction is less than 1.

(e) A fraction has a value greater than 1 if the numerator is larger than the denominator. For the fraction $2x/x$, the actual values of the numerator and the denominator will depend on the value of x. However, the numerator is twice as great as the denominator so that for any value of x (except zero), the numerator will be larger than the denominator. Therefore, the value of the fraction $2x/x$ will be greater than 1.

(i) One liter is 1000 mL. Therefore, the numerator, 100 mL, is smaller than the denominator, and the value of the fraction is less than 1.

2.26(a)

$$\% \text{ iron} = \frac{\text{g iron}}{\text{g compound}} \times 100\%$$

$$= \frac{2.33 \text{ g iron}}{3.00 \text{ g compound}} \times 100\% = 77.7\% \text{ iron}$$

2.28(c) The problem can be solved by proportions, by using the equation for percent, or by fractions.
By proportions,

$$\frac{\text{g oxygen}}{150 \text{ g compound}} = \frac{46.0 \text{ g oxygen}}{100 \text{ g compound}}$$

$$\text{g oxygen} = \frac{46.0 \text{ g oxygen}}{100 \text{ g compound}} \times 150 \text{ g compound}$$

$$= 69.0 \text{ g}$$

By the use of the equation for percent,

$$46.0\% \text{ oxygen} = \frac{\text{g oxygen}}{150 \text{ g compound}} \times 100\%$$

$$\text{g oxygen} = \frac{46.0\% \times 150 \text{ g}}{100\%} = 69.0 \text{ g}$$

By fractions,

$$46.0\% \text{ means } \frac{46.0}{100} \text{ of the total}$$

$$\text{g oxygen} = \frac{46.0}{100} \times 150 \text{ g compound} = 69.0 \text{ g}$$

2.21(a) As in Problem 2.27, the problem can be solved three ways.

$$95.0\% \; H_2SO_4 = \frac{5.60 \text{ g } H_2SO_4}{x \text{ g mixture}} \times 100\%$$

$$x = \frac{5.60 \text{ g } H_2SO_4}{95.0\% \; H_2SO_4} \times 100\% = 5.89 \text{ g mixture}$$

or

$$\frac{5.60 \text{ g } H_2SO_4}{x \text{ g mixture}} = \frac{95.0 \text{ g } H_2SO_4}{100 \text{ g mixture}}$$

$$x = \frac{5.60 \text{ g } H_2SO_4 \times 100 \text{ g mixture}}{95.0 \text{ g } H_2SO_4} = 5.89 \text{ g mixture}$$

or

$$95.0\% \text{ means that } \frac{95.0}{100} \text{ of the total is } H_2SO_4$$

$$5.60 \text{ g } H_2SO_4 = \frac{9.50}{100} \times \text{ g mixture}$$

$$\text{g mixture} = \frac{5.60 \text{ g} \times 100}{95.0} = 5.89 \text{ g}$$

CHAPTER 3

EXPONENTIAL NOTATION

3.1. EXPONENTIAL NOTATION

In scientific work it is frequently necessary to work with very large or very small numbers, such as 602,000,000,000,000,000,000,000 or 0.0000000008. Such numbers are difficult to write and to read accurately. Therefore, scientists use a notation that gives the numerical part of the answer and, separately, shows the position of the decimal point or the number of zeros by writing a power of 10. In this notation the numbers above become 6.02×10^{23} and 8×10^{-10}, respectively.

To see how this works, consider the number 500. This can be written as 5×100. But 100 can be written as 10×10 or 10^2. Therefore, 500 could be written as 5×10^2.

To understand the rules for writing numbers in exponential notation and for doing calculations with such numbers, it is necessary to be aware of the general rules for work with exponential numbers:

$$x^a (x^b) = x^{a+b}$$

To multiply exponentials, add the exponents.

$$\frac{x^a}{x^b} = x^{a-b}$$

To divide exponentials, subtract the exponents.

$$(x^a)^b = x^{ab}$$

To raise an exponential to a power, multiply the exponent.

$\sqrt[b]{x^a} = x^{a/b}$ **To find the root of an exponential, divide the exponent. (If $a = 1$, $\sqrt[b]{a} = a^{1/b}$.)**

■ EXAMPLE 1

$$10^2\,(10^3) = 10^{2+3} = 10^5$$

$$(10 \times 10)(10 \times 10 \times 10) = (10 \times 10 \times 10 \times 10 \times 10)$$

$$100{,}000 \equiv 100{,}000$$

Adding the exponents adds the number of times the base is used as a factor. ■

■ EXAMPLE 2

$$\frac{10^3}{10^1} = 10^{(3-1)} = 10^2$$

$$\frac{1000}{10} = 100$$

We know that any number divided by itself gives a quotient of 1. In exponential notation (for any value of x except 0),

$$\frac{x^a}{x^a} = x^{(a-a)} = x^0$$

By the rule that things equal to the same thing are equal to each other,

since $\frac{x^a}{x^a} = 1$ and $\frac{x^a}{x^a} = x^0$

then $x^0 = 1$

Conversely, it is useful to write 1 as x^0 for use in calculations.

Let us use this to see how to write 1/100 in exponential notation.

$$\frac{1}{100} = \frac{10^0}{10^2} = 10^{(0-2)} = 10^{-2}$$

Using decimal notation, dividing 1 by 100 gives 0.01. Therefore,

$$\frac{1}{100} = 0.01 = 10^{-2}$$ ■

$1{,}000{,}000 = 10^6$	(six zeros)
$100{,}000 = 10^5$	
$10{,}000 = 10^4$	
$1000 = 10^3$	
$100 = 10^2$	
$10 = 10^1$	(one zero)
$1 = 10^0$	(no zeros)
$0.1 = 10^{-1}$	(one place after the decimal)
$0.01 = 10^{-2}$	
$0.001 = 10^{-3}$	
$0.000001 = 10^{-6}$	(six places after the decimal)

Two patterns emerge:

For numbers greater than 1, the exponent shows the number of zeros after the number.

For numbers less than 1, the exponent shows the number of decimal places after the decimal point up to and including the first digit that is not zero.

Furthermore, the larger the *positive* exponent, the *larger* the number. The larger the *negative* exponent, the *smaller* the number.

3.2. WRITING NUMBERS IN EXPONENTIAL NOTATION

A number written in exponential notation has two parts. There is a *coefficient*, the numerical part, followed by 10 raised to some power, the *exponential* part. **In scientific notation, the numerical part is written with one non-zero digit before the decimal point.** That is, the number 525 is written as 5.25×10^2 rather than as 52.5×10^1 or 0.525×10^3. This is in some ways an arbitrary choice, and, in fact, the other ways are sometimes convenient for special purposes.

There are two general approaches to writing numbers in exponential form. One method is to rewrite the number as a product of a number (the coefficient) and a power of 10. Then convert the power-of-10 factor into exponential form.

EXAMPLE 3

Write the number 203,000 in exponential form with three significant figures.

Rewrite the number so that it has one digit before the decimal, times an exact power of 10.

$$203{,}000 = 2.03 \times 100{,}000$$

Then, since $100{,}000 = 10^5$,

$$203{,}000 = 2.03 \times 100{,}000 = 2.03 \times 10^5$$

The other approach is to count the number of places the decimal must be moved from its original position. The power of 10 shows how many places the decimal was moved. If the decimal is moved to the left, effectively making the number smaller, the exponent must be positive, effectively bringing the number back up to its original size. If the decimal is moved to the right, effectively making the number larger, the exponent must be negative, effectively making the number smaller again.

EXAMPLE 4

Use the counting-places procedure to convert 203,000 to exponential notation.

$$2\,0\,3\,0\,0\,0 = 2.03 \times 10^5$$

The decimal is moved five places to the left, so the exponent is $+5$.

EXAMPLE 5

Write 0.0098 in exponential notation.

Using the first procedure, we find that

$$\begin{aligned} 0.0098 &= 9.8 \times 0.001 \\ &= 9.8 \times 10^{-3} \end{aligned}$$

Using the second procedure, we obtain

$$0.\,0\,0\,9\,8 = 9.8 \times 10^{-3}$$

Remember that the decimal must go after the 9, to give one place before the point. The decimal is moved three places, so the exponent is -3. Since moving the decimal made the number larger, the exponent had to be negative to bring it down to the original size.

It does not matter which procedure is used for finding the size of the exponent; use whichever you find more convenient. Remember that you must not change the total size of the number while changing the notation.

A number like 3×10^{-4} is bigger than 10^{-4} but smaller than 10^{-3}. Write them out:

$$0.001 > 0.0003 > 0.0001 \qquad (> \text{means "greater than"})$$

The use of exponential notation makes it possible to show the number of significant figures unambiguously.

EXAMPLE 6

Express each in exponential notation, with three significant figures: 3750, 0.500.

$3750 = 3.75 \times 1000 = 3.75 \times 10^3$ Move the decimal three places to the left.

$0.500 = 5.00 \times 0.01 = 5.00 \times 10^{-1}$ Move the decimal one place to the right. ■

PROBLEMS

3.1 Write in exponential notation.
- (a) 2000 (b) 0.002
- *(c) 345,000 *(d) 0.000345
- (e) 5 billion (5,000,000,000)
- (f) 0.0000075 (g) 27
- (h) 932 (i) 0.002973

3.2 Write in decimal notation.
- *(a) 3×10^{-5} (b) 7.2×10^{-3}
- (c) 5×10^{-7} (d) 9.1×10^{6}
- *(e) 8.2×10^{0} (f) 2.98×10^{2}
- (g) 2.98×10^{-2} (h) 3.79×10^{1}

3.3 Multiply. Do not use a calculator.
- *(a) $10^2\,(10^2)$ *(b) $10^2\,(10^{-2})$
- (c) $10^5\,(10^7)$ (d) $10^5\,(10^{-7})$
- (e) $10^{-5}\,(10^7)$ *(f) $10^9\,(10^{-27})(10^6)$
- (g) $10^3\,(10^{12})(10^{-16})$ (h) $10^{25}\,(10^{102})$
- (i) $x^3\,(x^5)$ (j) $x^m\,(x^n)$

3.4 Divide. Do not use a calculator.

*(a)	$10^2 \div 10^3$	(b)	$10^3 \div 10^2$
(c)	$10^9 \div 10^{27}$	(d)	$10^{16} \div 10^9$
*(e)	$10^7 \div 10^{-16}$	(f)	$10^{16} \div 10^{-9}$
(g)	$10^{-16} \div 10^9$	(h)	$10^{150} \div 10^{30}$
(i)	$e^H \div e^T$	(j)	$x^m \div x^n$

3.3. CALCULATIONS WITH NUMBERS IN EXPONENTIAL NOTATION

3.3.A. Multiplication and Division of Numbers in Exponential Notation

Multiplication and division of numbers expressed in exponential notation is accomplished by multiplying the coefficients (numerical portions) and separately multiplying the exponential parts of the numbers. Where no coefficient is shown, a coefficient of 1 is implied.

EXAMPLE 7

Multiply 3.0×10^5 by 1.7×10^{-2}.

First, separate the coefficients from the exponentials. Group the coefficients and separately group the exponentials. Multiply each.

$$\begin{aligned}(3.0 \times 10^5) \times (1.7 \times 10^{-2}) &= (3.0 \times 1.7) \times (10^5 \times 10^{-2}) \\ &= 5.1 \times 10^3\end{aligned}$$

EXAMPLE 8

Multiply 5.9×10^{17} by 4.6×10^{-9}.

$$\begin{aligned}(5.9 \times 10^{17}) \times (4.6 \times 10^{-9}) &= (5.9 \times 4.6) \times (10^{17} \times 10^{-9}) \\ &= 27 \times 10^8\end{aligned}$$

Here 27 has two digits before the decimal, so the answer is not in the proper form. To correct it, write the coefficient 27 in proper exponential form and multiply by the exponential part.

$$27 \times 10^8 = (2.7 \times 10^1) \times 10^8 = 2.7 \times 10^9$$

Compare this calculation with the same multiplication problem written in decimal form: $590{,}000{,}000{,}000{,}000{,}000 \times 0.0000000046$. The use of the exponential form helps a great deal to place the decimal correctly.

Division is accomplished in the same way as multiplication but by dividing the coefficients and dividing the exponentials.

■ **EXAMPLE 9**

Divide 8.4×10^{-4} by 2.0×10^{-8}.

$$\frac{8.4 \times 10^{-4}}{2.0 \times 10^{-8}} = \frac{8.4}{2.0} \times \frac{10^{-4}}{10^{-8}}$$
$$= 4.2 \times 10^{-4-(-8)}$$
$$= 4.2 \times 10^{4}$$
■

■ **EXAMPLE 10**

Divide 1.8×10^{5} by 7.2×10^{7}.

$$\frac{1.8 \times 10^{5}}{7.2 \times 10^{7}} = \frac{1.8}{7.2} \times \frac{10^{5}}{10^{7}}$$
$$= 0.25 \times 10^{-2}$$

Write this in proper form.

$$0.25 \times 10^{-2} = (2.5 \times 10^{-1}) \times 10^{-2}$$
$$= 2.5 \times 10^{-3}$$
■

When dividing, it is sometimes more convenient to use a procedure that corrects the position of the decimal point before, rather than after, the arithmetic is done. To accomplish this, start out by rewriting the numerator in a form that makes the coefficient larger than that of the denominator, but not as much as 10 times as large. Then the coefficient of the quotient will come out to be between 1 and 10. Although this requires writing the number of the numerator with more than one digit before the decimal, the numbers may be written in any convenient form for calculations.

$$\frac{1.8 \times 10^{5}}{7.2 \times 10^{7}} = \frac{(18 \times 10^{-1}) \times 10^{5}}{7.2 \times 10^{7}}$$
$$= \frac{18}{7.2} \times \frac{10^{4}}{10^{7}}$$
$$= 2.5 \times 10^{-3}$$

EXAMPLE 11

Divide 10^{-14} by 2.5×10^{-5}.

$$\frac{10^{-14}}{2.5 \times 10^{-5}} = \frac{1 \times 10^{-14}}{2.5 \times 10^{-5}}$$
$$= \frac{10 \times 10^{-15}}{2.5 \times 10^{-5}}$$
$$= 4.0 \times 10^{-10}$$

For most people it is easier to calculate 10/2.5 than 1/2.5, so moving the decimal at the start results in an easier calculation as well as correct placement of the decimal in the answer. ■

Several multiplication and division steps can be combined.

EXAMPLE 12

Solve.

$$\frac{(2.1 \times 10^{2})(9.8 \times 10^{-3})(5.3 \times 10^{9})}{(8.7 \times 10^{-5})(3.6 \times 10^{10})}$$

Separating the coefficients from the exponentials, we write

$$\frac{2.1 \times 9.8 \times 5.3}{8.7 \times 3.6} \times \frac{10^{2-3+9}}{10^{-5+10}} = 3.5 \times 10^{3}$$

The arithmetic can be done in any sequence of multiplication and division steps. It is probably best for a beginner to multiply all the factors in the numerator and, separately, multiply all the factors in the denominator, then divide.

$$\frac{2.1 \times 9.8 \times 5.3}{8.7 \times 3.6} = \frac{109}{31} = 3.5$$

Whatever sequence of steps is used, it is essential to divide by both factors that appear in the denominator. ■

EXAMPLE 13

Perform the calculation.

$$\frac{0.68 \times 34.6}{0.082 \times 303}$$

Although this can be done directly, it is better to rewrite it in exponential form to make sure of the location of the decimal point. Then the numerical calculation can be performed rapidly.

$$\frac{(6.8 \times 10^{-1}) \times (3.46 \times 10^{1})}{(8.2 \times 10^{-2}) \times (3.03 \times 10^{2})} = \frac{6.8 \times 3.46}{8.2 \times 3.03} \times \frac{10^{-1+1}}{10^{-2+2}}$$

$$= 0.95 \times 10^{0}$$

$$= 9.5 \times 10^{-1}$$

■

■ EXAMPLE 14

Find the value of K in the equation if $[H^+] = 3.6 \times 10^{-6}$, $[A^-] = 4.0 \times 10^{-3}$, and $[HA] = 6.0 \times 10^{-2}$.

$$K = \frac{[H^+][A^-]}{[HA]}$$

[The symbol for a chemical substance enclosed in square brackets indicates that the concentration of the substance is to be used as the factor. Unless otherwise indicated, the unit of concentration is the molarity, M (see Section 5.3), so $[H^+]$ is read "the molarity of hydrogen ion."]

First, substitute the numerical values for their symbols in the equation. Then, perform the calculation.

$$K = \frac{(3.6 \times 10^{-6})(4.0 \times 10^{-3})}{6.0 \times 10^{-2}}$$

$$= \frac{3.6\ (4.0)}{6.0} \times \frac{10^{-6}\ (10^{-3})}{10^{-2}}$$

$$= 2.4 \times 10^{-7}$$

■

PROBLEMS

3.5 Multiply.

(a) $(2 \times 10^{3})(4 \times 10^{6})$
(b) $(2 \times 10^{3})(4 \times 10^{-6})$
*(c) $(8.0 \times 10^{3})(4.0 \times 10^{9})$
(d) $(3.0 \times 10^{7})(4.0 \times 10^{-6})$
*(e) $37\ (2.0 \times 10^{-8})$
(f) $1.5\ (6.0 \times 10^{23})$
(g) $9.0\ (5.0 \times 10^{-3})$
(h) $(7.5 \times 10^{-9})(3.2 \times 10^{4})$
(i) $(6.2 \times 10^{-5})(6.2 \times 10^{-5})$

3.6 Divide.

(a) $(4 \times 10^6) \div (2 \times 10^3)$
*(b) $(2 \times 10^3) \div (4 \times 10^{-6})$
(c) $(6.0 \times 10^{-5}) \div (8.0 \times 10^7)$
(d) $(8 \times 10^3) \div (4 \times 10^{-6})$
(e) $(9.6 \times 10^9) \div (3.2 \times 10^{-8})$
*(f) $27 \div (9 \times 10^{-9})$
(g) $31.4 \div (6.02 \times 10^{23})$
*(h) $10^{-14} \div (2.5 \times 10^{-9})$
(i) $10^9 \div (5 \times 10^3)$
(j) $10^{-9} \div (2 \times 10^6)$

3.7 Calculate.

*(a) $\dfrac{(5.0 \times 10^7)(3.0 \times 10^{-9})}{2.0 \times 10^3}$
(b) $\dfrac{(3.2 \times 10^4)(2 \times 10^3)}{16 \times 10^{-3}}$

(c) $\dfrac{(2 \times 10^2)(6 \times 10^3)}{(8 \times 10^{-2})(3 \times 10^2)}$
*(d) $\dfrac{270 \times 2800}{0.009 \times 120}$

(e) $\dfrac{1900 \times 0.04}{3.8 \times 5000}$
(f) $\dfrac{760 \times 300 \times 450}{1520 \times 275}$

3.8 Substitute each set of values of a, b, and c into the equation $x = ab/c$, and calculate the value of x.

(a) $a = 7.60 \times 10^2$ $b = 1.20 \times 10^2$ $c = 2.73 \times 10^2$
(b) $a = 5.0 \times 10^9$ $b = 3.2 \times 10^{-3}$ $c = 5.0 \times 10^3$
(c) $a = 3.0 \times 10^{-5}$ $b = 6.0 \times 10^{23}$ $c = 1.0 \times 10^3$
(d) $a = 8.6 \times 10^5$ $b = 1.9 \times 10^{-1}$ $c = 6.0 \times 10^{-5}$
(e) $a = 4.7 \times 10^{-3}$ $b = 9.2 \times 10^{-5}$ $c = 2.1 \times 10^4$
(f) $a = 1.3 \times 10^{-2}$ $b = 5.2 \times 10^{-7}$ $c = 6.3 \times 10^{-9}$

3.9 Calculate n in the equation

$$n = \frac{PV}{RT}$$

for each set of values of P, V, R, and T. Write the answer with the proper units.

(a) $P = 2.50 \times 10^2$ Torr; $V = 6.91 \times 10^{-1}$ L;

$$R = 6.24 \times 10^1 \frac{\text{L Torr}}{\text{mole K}} \qquad T = 3.73 \times 10^2 \text{ K}$$

(b) $P = 9.6 \times 10^{-1}$ atm; $V = 7.7 \times 10^{-2}$ L;

$$R = 8.2 \times 10^{-2} \frac{\text{L atm}}{\text{mole K}} \qquad T = 2.98 \times 10^2 \text{ K}$$

3.10 Calculate K for each, given the equations and values shown.

*(a) $K_{sp} = [Ag^+][Cl^{-1}]$; $[Ag^+] = [Cl^-] = 1.26 \times 10^{-5}$
(b) $K_{sp} = [Ag^+][Cl^-]$; $[Ag^+] = 1.6 \times 10^{-2}$ and $[Cl^-] = 1.0 \times 10^{-8}$

(c) $K_{sp} = [Zn^{2+}][S^{2-}]$; $[Zn^{2+}] = 1.0 \times 10^{-2}$
and $[S^{2-}] = 1.2 \times 10^{-21}$

(d) $K_a = \dfrac{[H^+][F^-]}{[HF]}$; $[H^+] = [F^-] = 6.0 \times 10^{-3}$

and $[HF] = 1.0 \times 10^{-1}$

(e) $K_a = \dfrac{[H^+][F^-]}{[HF]}$; $[H^+] = 1.0 \times 10^{-5}$, $[F^-] = 3.5 \times 10^{-1}$,

and $[HF] = 10^{-2}$; compare the result with that of 3.10(d)

(f) $K_a = \dfrac{[H^+][HPO_4^{2-}]}{[H_2PO_4^-]}$; $[H^+] = 1 \times 10^{-6}$, $[HPO_4^{2-}] =$
6×10^{-5}, and $[H_2PO_4^-] = 1 \times 10^{-3}$

[*Caution:* Be careful to substitute correctly.]

3.3.B. Powers and Roots in Exponential Notation

When a number written in exponential notation is to be raised to a power, both the coefficient and the exponential must be raised to the power.

$$(2 \times 10^2)^3 = 2^3 \times (10^2)^3$$

Raising a number to the third power means using it as a factor three times.

$$2^3 = 2 \times 2 \times 2 = 8$$

$$(10^2)^3 = 10^2 \times 10^2 \times 10^2 = 10^{(2+2+2)} = 10^6$$

This last step can be summarized by the general rule that to raise an exponential to a power, multiply the exponents.

$$(10^2)^3 = 10^{(3 \times 2)} = 10^6$$

Therefore,

$$(2 \times 10^2)^3 = 2^3 \times (10^2)^3 = 8 \times 10^6$$

EXAMPLE 15

Calculate the value of $(3 \times 10^5)^4$:

$$(3 \times 10^5)^4 = 3^4 \times (10^5)^4 = 81 \times 10^{20}$$

The position of the decimal must be corrected so there is only one nonzero digit before the decimal point.

$$81 \times 10^{20} = 8.1 \times 10^{1} \times 10^{20} = 8.1 \times 10^{21}$$

EXAMPLE 16

Given the equation $K_{sp} = [Cd^{2+}][OH^-]^2$, calculate K_{sp} if $[Cd^{2+}] = 7.5 \times 10^{-6}$ and $[OH^-] = 4.0 \times 10^{-5}$.

Substitute the values into the equation and multiply.

$$\begin{aligned} K_{sp} &= (7.5 \times 10^{-6})(4.0 \times 10^{-5})^2 \\ &= (7.5 \times 10^{-6})(4.0)^2 (10^{-5})^2 \\ &= (7.5 \times 16)(10^{-6} \times 10^{-10}) \\ &= 120 \times 10^{-16} \end{aligned}$$

Adjust the position of the decimal.

$$120 \times 10^{-16} = (1.2 \times 10^2)(10^{-16})$$

Therefore,

$$K_{sp} = 1.2 \times 10^{-14}$$

PROBLEMS

3.11 (a) $(10^2)^6$ *(b) $(10^{-2})^6$
(c) $(10^{-6})^{-2}$ (d) $(10^4)^3$
*(e) $(10^{1/2})^4$ (f) $(e^2)^2$
(g) $(x^5)^3$ (h) $(x^{-3})^5$

3.12 (a) $(3 \times 10^9)^2$ (b) $(3 \times 10^{-9})^2$
*(c) $(7 \times 10^2)^3$ (d) $(2 \times 10^{-4})^5$
(e) $(9 \times 10^7)^2$

3.13 Substitute the numerical values for a, b, and c into the equations, and calculate x and y.

$$x = ab^2 \qquad y = \frac{ab^3}{c^2}$$

(a) $a = 1.0 \times 10^2$ $b = 2.0 \times 10^4$ $c = 2.0 \times 10^3$
*(b) $a = 5.0 \times 10^{-2}$ $b = 2.0 \times 10^3$ $c = 4.0 \times 10^4$

(c) $a = 1.0 \times 10^{-3}$ $b = 2.0 \times 10^{-2}$ $c = 2.0 \times 10^{4}$
(d) $a = 2.0 \times 10^{5}$ $b = 5.0 \times 10^{-9}$ $c = 1.2 \times 10^{-8}$

3.14 Find each K_{sp} given the equations and values shown.
*(a) $K_{sp} = [Pb^{2+}][I^-]^2$; $[Pb^{2+}] = 1.0 \times 10^{-2}$ and $[I^-] = 1.2 \times 10^{-3}$
(b) Use the same equation as (a) but $[Pb^{2+}] = 1.44 \times 10^{-4}$ and $[I^-] = 10^{-2}$. Compare the results with the answer to (a).
(c) $K_{sp} = [Ca^{2+}][F^-]^2$; $[Ca^{2+}] = 2.0 \times 10^{-3}$ and $[F^-] = 1.3 \times 10^{-4}$
(d) $K_{sp} = [Ag^+]^2[S^{2-}]$; $[Ag^+] = 1.3 \times 10^{-14}$ and $[S^{2-}] = 1.0 \times 10^{-21}$
(e) $K_{sp} = [Al^{3+}][OH^-]^3$; $[Al^{3+}] = 3.7 \times 10^{-3}$ and $[OH^-] = 1.0 \times 10^{-4}$

The rule for finding a root of an exponential is to divide the exponent, exactly the inverse of the multiplication done in raising to a power.

$$\sqrt[n]{x^a} = x^{a/n}$$
$$\sqrt[2]{10^6} = 10^{6/2} = 10^3$$

Because this is the procedure, roots are commonly and conveniently written as fractional powers. That is, the square root is the 1/2 power, the cube root is the 1/3 power, and so on.

$$\sqrt[n]{x} = x^{1/n}$$

If the original exponent is not an even multiple of the root index, n, the root would have a fractional exponent. The square root of 10^5 is thus $10^{5/2}$.

To find the root of a number expressed in exponential notation, reverse the process of raising to a power. Find the root of the coefficient and that of the exponential; the product of the roots is the root of the original number.

$$\sqrt[3]{8 \times 10^9} = \sqrt[3]{8} \times \sqrt[3]{10^9} = 8^{1/3} \times (10^9)^{1/3}$$
$$= 2 \times 10^3$$

If you are not using a calculator, you can find the root of the exponential portion only when the exponent is an even multiple of the root.

If the exponent is not an even multiple of the root, the number must be rewritten to make the exponent an even multiple of the root and the coefficient a number greater than 1.

EXAMPLE 17

Find the square root of 1.0×10^5.

$$\sqrt{1.0 \times 10^5} = (1.0 \times 10^5)^{1/2} = (10 \times 10^4)^{1/2} \qquad \text{not } (0.10 \times 10^6)^{1/2}$$
$$= (10)^{1/2} \times (10^4)^{1/2}$$
$$= \pm 3.2 \times 10^2$$

EXAMPLE 18

Find the fifth root of 8.1×10^{-16}.

To make the exponent an even multiple of 5, you could change it to either 10^{-15} or 10^{-20}.

$$8.1 \times 10^{-16} = 8.1\,(10^{-1} \times 10^{-15}) = 0.81 \times 10^{-15}$$

or

$$8.1 \times 10^{-16} = 8.1\,(10^4 \times 10^{-20}) = 81{,}000 \times 10^{-20}$$

Use the one that gives a coefficient greater than 1.

$$(8.1 \times 10^{-16})^{1/5} = (81{,}000 \times 10^{-20})^{1/5}$$
$$= 9.6 \times 10^{-4}$$

The fifth root of 81,000 can be found using logarithms (Chapter 8). Some calculators have functions that can be used to find roots.

PROBLEM

3.15 Find the root.

*(a) $\sqrt{10^6}$ (b) $\sqrt[3]{10^{-6}}$ (c) $\sqrt[5]{10^{15}}$
*(d) $\sqrt[2]{1.0 \times 10^9}$ (e) $\sqrt[3]{1.00 \times 10^{13}}$ (f) $\sqrt[3]{505}$
*(g) $\sqrt[4]{0.0016}$ (h) $\sqrt[2]{0.16}$ (i) $\sqrt[4]{5.2 \times 10^{-9}}$
(j) $\sqrt[3]{2.7 \times 10^{11}}$

3.3.C. Addition and Subtraction

The addition and subtraction of numbers expressed in exponential form require that the exponents be the same for the numbers to be added or subtracted.

The procedure in the decimal system is to line up the decimal places and add the corresponding columns.

$$\begin{array}{r} 4500 \\ +\ 250 \\ +\ \underline{\quad 9} \\ 4759 \end{array}$$

Remember that each column represents a power of 10. The first place to the left of the decimal contains the numbers that multiply the zeroth power of 10, the ones. The next column is called the tens column and contains the numbers multiplying 10^1, the hundreds column contains the numbers multiplying 10^2, and so on. In the addition process, numbers in the tens column can only be added to numbers in the tens column, never to numbers in the ones or hundreds column. It is not correct to say that $20 + 3 = 50$, using the 3 in the tens column instead of the ones column. (You already know this but never think about it.) The same rule applies when numbers are written as exponentials. Numbers to be added or subtracted must be in the same column or, in other words, have the same power of 10. **Only the coefficients are added. The exponential term defines the position of the decimal point, which does not change in addition.**

EXAMPLE 19

Add 2.70×10^3 and 3.3×10^2.

Before the numbers can be added, they must have the same exponent. The problem can be rephrased either with both numbers using 10^2, or with both using 10^3.

$$2.70 \times 10^3 = 2.70 \times (10^1 \times 10^2)$$

$$= (2.70 \times 10^1) \times 10^2 = 27.0 \times 10^2$$

$$(27.0 \times 10^2) + (3.3 \times 10^2) = 30.3 \times 10^2 = 3.03 \times 10^3$$

or

$$3.3 \times 10^2 = 3.3 \times (10^{-1} \times 10^3)$$

$$= (3.3 \times 10^{-1}) \times 10^3 = 0.33 \times 10^3$$

$$(0.33 \times 10^3) + (2.70 \times 10^3) = 3.03 \times 10^3$$

EXAMPLE 20

Subtract 344×10^{-7} from 5.05×10^{-5}.

Working in powers of 10^{-7}, we can write

$$5.05 \times 10^{-5} = 5.05 \times (10^2 \times 10^{-7})$$ The quantities in parentheses equal the original 10^{-5}.

$$= (5.05 \times 10^2) \times 10^{-7}$$

$$= 505 \times 10^{-7}$$

$$(505 \times 10^{-7}) - (344 \times 10^{-7}) = 161 \times 10^{-7}$$

or, in powers of 10^{-5}:

$$344 \times 10^{-7} = 344 \times (10^{-2} \times 10^{-5})$$
$$= (344 \times 10^{-2}) \times 10^{-5} = 3.44 \times 10^{-5}$$

Then

$$(5.05 \times 10^{-5}) - (3.44 \times 10^{-5}) = 1.61 \times 10^{-5}$$

For Examples 19 and 20, try writing the problem in decimal notation. ■

PROBLEM

3.16 Add or subtract as indicated.
*(a) $5.02 \times 10^3 + 3.79 \times 10^2$
(b) $1.95 \times 10^4 + 6.81 \times 10^6$
(c) $1.95 \times 10^{-4} - 6.81 \times 10^{-6}$
(d) $1.100 \times 10^2 - 1.000 \times 10^{-1}$
(e) $3.285 \times 10^{-3} + 7.02 \times 10^{-1}$
(f) $4.95 \times 10^4 + 2.01 \times 10^{-1}$
(g) $2.4 \times 10^{-3} + 3.2 \times 10^{-2}$

3.4. EXPONENTIAL NUMBERS ON A CALCULATOR

If you have a calculator that does not include a function for exponential numbers, separate each number in a problem into the coefficients and the exponential parts. Use the calculator for the computation involving the coefficients. Then do the calculation for the exponential part of the problem as needed. Finally, correct the position of the decimal point if necessary so there is one digit before the decimal point.

■ EXAMPLE 21

Perform the calculation without using a calculator for the exponential part.

$$\frac{3.20 \times 10^3 (3.68 \times 10^{-5})}{1.90 \times 10^6 (9.71 \times 10^{-2})} = \frac{3.20 \times 3.68}{1.90 \times 9.71} \times \frac{10^3 \times 10^{-5}}{10^6 \times 10^{-2}}$$
$$= 0.638 \times 10^{-6}$$
$$= 6.38 \times 10^{-1} \times 10^{-6}$$
$$= 6.38 \times 10^{-7}$$

Be careful to divide by both numbers in the denominator. The calculator display reads 0.6383001, but the numbers in the problem have only three significant figures, so the answer must be given to three significant figures.

If you have a calculator that is capable of using exponential notation, the calculator will show the coefficient, followed by the exponent. If the exponent is positive, there will be a space between them. If the exponent is negative, there will be a minus sign between the numbers.

5.12×10^7	appears as	5 . 1 2 7
9.738×10^{-12}	appears as	9 . 7 3 8 − 1 2
10^{-3}	appears as	1 − 3 (1×10^{-3})

Notice that a coefficient is *always* shown and the 10 is not shown.

Be sure you learn how to enter exponential numbers correctly on *your* calculator. Usually, you must enter the coefficient, press the key (marked EE on many calculators, EXP on others) that tells the calculator you are using exponential notation, and then enter the exponent. If the number is negative, press the ± key after entering the coefficient to change the sign of the number. If the exponent is negative, press the ± key after entering the exponent. ■

■ EXAMPLE 22

Enter each number on your calculator.

-5.12×10^7	Press [5] [.] [1] [2] [±] [EE] [7] Register shows −5.12 7.
9.738×10^{-12}	Press [9] [.] [7] [3] [8] [EE] [1] [2] [±].
10^{-3}	You must enter a coefficient; if no coefficient is shown, the coefficient 1 is always implied. Therefore rewrite:
1×10^{-3}	Press [1] [EE] [3] [±]. ■

Even when you can use your calculator, as with any other computation you should do an approximate mental calculation to see if the answer is reasonable. You should know what sort of answer you ought to get, so you can spot the problem if you entered numbers incorrectly or if there is some other error in doing the calculation. A calculator cannot handle exponents greater than 99. Sometimes it is possible to do a calculation in your head, without bothering with the calculator.

SOLUTIONS TO STARRED PROBLEMS

3.1(c) To write 345,000 in exponential notation,

$$345{,}000 = 3.45 \times 100{,}000 = 3.45 \times 10^5$$

since 100,000 has five zeros and is

$$10 \times 10 \times 10 \times 10 \times 10 \qquad \text{or} \qquad 3\underbrace{,4\,5\,0\,0\,0}_{} = 3.45 \times 10^5$$

since the decimal was moved five places to make the number smaller, the exponent must make it five places larger.

(d) To write 0.000345 in exponential notation, write

$$0.000345 = 3.45 \times 0.0001 = 3.45 \times 10^{-4}$$

Write the coefficient with one digit before the decimal point. For the exponential, count the number of places after the decimal point, including the first digit. Or write

$$0\underbrace{.\,0\,0\,0\,3}_{},4\,5 = 3.45 \times 10^{-4}$$

To get one digit before the decimal, the point was moved four places, making the number larger. The exponent must make it smaller again by four places.

3.2(a) The digit 3 must be in the fifth place after the decimal:

$$0.00003$$

(e) Since $10^0 = 1$,

$$8.2 \times 10^0 = 8.2 \times 1 = 8.2$$

3.3(a) To multiply, add the exponents.

$$10^2\,(10^2) = 10^{2+2} = 10^4$$

(b) Adding the exponents, we find that

$$10^2\,(10^{-2}) = 10^{2-2} = 10^0 = 1$$

(f) Add all the exponents.

$$10^9 (10^{-27})(10^6) = 10^{9-27+6} = 10^{-12}$$

3.4(a) To divide, subtract the exponents.

$$\frac{10^2}{10^3} = 10^{2-3} = 10^{-1}$$

(e) Subtracting a negative number requires changing the sign and adding.

$$\frac{10^7}{10^{-16}} = 10^{7-(-16)} = 10^{7+16} = 10^{23}$$

3.5(c) Multiply the coefficients and multiply the exponentials.

$$(8.0 \times 10^3)(4.0 \times 10^9) = (8.0 \times 4.0)(10^3 \times 10^9) = 32 \times 10^{12}$$

To adjust the position of the decimal, rewrite 32 as 3.2×10^1. Then

$$32 \times 10^{12} = 3.2 \times 10^1 \times 10^{12} = 3.2 \times 10^{13}$$

(e) The problem can be handled in one of two ways.

$$37 \times 2.0 \times 10^{-8} = (37 \times 2.0) \times 10^{-8} = 74 \times 10^{-8}$$

Then correct the decimal.

$$74 \times 10^{-8} = 7.4 \times 10^1 \times 10^{-8} = 7.4 \times 10^{-7}$$

The other method is to start by writing all numbers in exponential form; then group and multiply:

$$\begin{aligned} 37 &= 3.7 \times 10^1 \\ 37\,(2.0 \times 10^{-8}) &= (3.7 \times 10^1)(2.0 \times 10^{-8}) \\ &= (3.7 \times 2.0)(10^1 \times 10^{-8}) \\ &= 7.4 \times 10^{-7} \end{aligned}$$

3.6(b) Divide the coefficients and divide the exponentials.

$$\frac{2 \times 10^3}{4 \times 10^{-6}} = \frac{2}{4} \times \frac{10^3}{10^{-6}} = 0.5 \times 10^{3-(-6)} = 0.5 \times 10^9$$

Then correct the position of the decimal.

$$0.5 \times 10^{9} = 5 \times 10^{-1} \times 10^{9} = 5 \times 10^{8}$$

An alternative procedure would be to correct the position of the decimal first by making the coefficient in the numerator larger than the coefficient in the denominator. To do this, convert 2 into 20×10^{-1}.

$$\frac{2 \times 10^{3}}{4 \times 10^{-6}} = \frac{20 \times 10^{-1} \times 10^{3}}{4 \times 10^{-6}} = 5 \times 10^{8}$$

(f) Consider 27 as 27×1, which is 27×10^{0}, or else write 27 in exponential form, 2.7×10^{1}.

$$\frac{27}{9 \times 10^{-9}} = \frac{27}{9} \times \frac{10^{0}}{10^{-9}} = 3 \times 10^{9}$$

or

$$\frac{2.7 \times 10^{1}}{9 \times 10^{-9}} = \frac{2.7}{9} \times \frac{10^{1}}{10^{-9}} = 0.3 \times 10^{10}$$

$$0.3 \times 10^{10} = 3 \times 10^{-1} \times 10^{10} = 3 \times 10^{9}$$

(h) Consider 10^{-14} as 1×10^{-14} or, for simpler calculation, as 10×10^{-15}.

$$\frac{1 \times 10^{-14}}{2.5 \times 10^{-9}} = 0.4 \times 10^{-5} = 4 \times 10^{-6}$$

or

$$\frac{10 \times 10^{-15}}{2.5 \times 10^{-9}} = 4 \times 10^{-6}$$

3.7(a) Group the coefficients and group the exponentials.

$$\frac{(5.0)(3.0)}{2.0} \times \frac{10^{7} \times 10^{-9}}{10^{3}} = \frac{15}{2.0} \times \frac{10^{-2}}{10^{3}}$$

$$= 7.5 \times 10^{-5}$$

(d) Rewrite the problem with the numbers in exponential form.

$$\frac{270 \times 2800}{0.009 \times 120} = \frac{2.7 \times 10^2 \times 2.8 \times 10^3}{9 \times 10^{-3} \times 1.2 \times 10^2}$$

Now group the coefficients and group the exponentials. Then do the calculation; if you simplify by canceling, you can save a considerable amount of arithmetic. In fact, you might wish to write 270 as 27×10^1 and cancel the 27 with the 9 leaving 3. Sometimes, as in Problem 3.7(f), it is more convenient to cancel before writing the numbers as exponentials. The aim is to get the right answer. Use whichever procedure looks most convenient.

$$\frac{2.7 \times 2.8}{9 \times 1.2} \times \frac{\cancel{10^2} \times 10^3}{10^{-3} \times \cancel{10^2}} = 0.7 \times 10^6 = 7 \times 10^5$$

The following cancellations were possible for the arithmetic. Portions of the fraction will be shown to make clear which parts are being canceled.

$\frac{2.7}{9} = 0.3$ dividing numerator and denominator by 9

$\frac{2.8}{1.2} = \frac{28}{12} = \frac{7}{3}$ dividing by 4

$\frac{0.3 \times 7}{3} = 0.7$ recombining the parts and dividing numerator and denominator by 3

3.10(a) First substitute the values given into the equation.

$$K_{sp} = [Ag^+][Cl^-] \qquad \text{and} \qquad [Ag^+] = [Cl^-] = 1.26 \times 10^{-5}$$

$$K_{sp} = (1.26 \times 10^{-5})(1.26 \times 10^{-5})$$

Now group the coefficients and group the exponentials, and calculate the product.

$$K_{sp} = (1.26 \times 1.26)(10^{-5} \times 10^{-5})$$
$$= 1.59 \times 10^{-10}$$

3.11(b) To raise an exponential to a power, multiply the exponents.

$$(10^{-2})^6 = 10^{-2 \times 6} = 10^{-12}$$

Remember that the product of a negative number and a positive number is negative.

(e) The exponents are multiplied, even if one of them is a fraction.

$$(10^{1/2})^4 = 10^{4/2} = 10^2$$

3.12(c) Both the coefficient and the exponential parts must be raised to the specific power.

$$(7 \times 10^2)^3 = 7^3 \times (10^2)^3$$
$$= 343 \times 10^6$$

Correct the position of the decimal.

$$343 \times 10^6 = (3.43 \times 10^2) \times 10^6$$
$$= 3.43 \times 10^8$$

3.13(b) Substitute the values of a and b into the expression for x, and perform the multiplication.

$$x = ab^2 = 5.0 \times 10^{-2}\,(2.0 \times 10^3)^2$$
$$= 5.0\,(2.0)^2 \times 10^{-2}\,(10^3)^2$$
$$= 5.0\,(4.0) \times 10^{-2+6}$$
$$= 20 \times 10^4$$

Adjust the position of the decimal point.

$$x = 20 \times 10^4 = 2.0 \times 10 \times 10^4 = 2.0 \times 10^5$$

Similarly, for y,

$$y = \frac{ab^3}{c^2} = \frac{5.0 \times 10^{-2}\,(2.0 \times 10^3)^3}{(4.0 \times 10^4)^2}$$
$$= \frac{5.0\,(2.0)^3}{(4.0)^2} \times \frac{10^{-2}\,(10^3)^3}{(10^4)^2} = \frac{5.0\,(8.0)}{16} \times \frac{10^{-2+9}}{10^8}$$
$$= 2.5 \times 10^{7-8} = 2.5 \times 10^{-1}$$

Since there is one digit before the decimal, no adjustment is necessary.

3.14(a) Substitute the values given into the equation; then do the calculation.

$$K_{sp} = [Pb^{2+}][I^-]^2;$$

$$[Pb^{2+}] = 1.0 \times 10^{-2} \qquad \text{and} \qquad [I^-] = 1.2 \times 10^{-3}$$

$$K_{sp} = 1.0 \times (10^{-2})(1.2 \times 10^{-3})^2$$

$$= (1.0)(10^{-2})(1.2)^2\,(10^{-3})^2$$ Both the 1.2 and the 10^{-3} must be squared.

$$= 1.4 \times 10^{-2} \times 10^{-6}$$

$$= 1.4 \times 10^{-8}$$

3.15(a) To find the root of a number expressed in exponential notation, divide the exponent.

$$\sqrt[2]{10^6} = (10^6)^{1/2} = 10^{6/2} = \pm 10^3$$

Since either a positive or a negative number raised to an even power gives a positive number, the root could be either positive or negative.

(d) Here the exponent, 9, is not evenly divisible by the index, 2. Therefore, the number must be rewritten with an exponent that is evenly divisible by 2. It would be possible to write either

$$\sqrt[2]{1.0 \times 10^9} = \sqrt[2]{10 \times 10^8} \qquad \text{or} \qquad \sqrt[2]{0.10 \times 10^{10}}$$

The first of these is used for two reasons: (1) the root will have the decimal in the right place, and (2) if you are looking up the square root in a table, you will find 10 (but not 0.1) listed.

$$\sqrt[2]{10 \times 10^8} = \sqrt[2]{10} \times \sqrt[2]{10^8}$$ This can be done for multiplication but *not* for addition.

$$= \pm 3.2 \times 10^4$$

(g) Rewrite the number in exponential form, being careful to use an exponent that is evenly divisible by the index, 4. This will, of course, require that you ignore the usual role of one digit before the decimal.

$$\sqrt[4]{0.0016} = \sqrt[4]{16 \times 10^{-4}} \qquad \text{rather than} \qquad \sqrt[4]{1.6 \times 10^{-3}}$$

To take the fourth root of 16, you can take the square root, and then take the square root of that square root.

$$(16)^{1/4} = (16^{1/2})^{1/2} = 4.0^{1/2} = 2.0$$

Since either a positive or a negative number would give a positive number when raised to an even power, there is no way of

knowing whether the root should be positive or negative. Therefore,

$$\sqrt[4]{16 \times 10^{-4}} = \pm 2.0 \times 10^{-1} = \pm 0.20$$

3.16(a) Before the numbers can be added, the exponents must be the same.

$$3.79 \times 10^2 = 0.379 \times 10^3$$

$$\begin{array}{r} 5.02 \times 10^3 = 5.02 \times 10^3 \\ +0.379 \times 10^3 = \underline{0.38 \times 10^3} \\ \hline 5.399 \times 10^3 = 5.40 \times 10^3 \end{array}$$

Since one of the numbers has its last digit in the second place after the decimal, the last digit in the answer must be in the second place after the decimal.

Alternatively,

$$5.02 \times 10^3 = 50.2 \times 10^2$$

$$50.2 \times 10^2 + 3.79 \times 10^2 = 54.0 \times 10^2 = 5.40 \times 10^3$$

CHAPTER 4

RAPID MENTAL CALCULATIONS

The importance of looking at your answer to a problem to see if it makes sense has been stressed repeatedly. This chapter discusses ways to (1) make a quick evaluation; (2) perform a rapid approximate calculation, so you can determine if the size of the answer is about right; and (3) simplify the arithmetic in a calculation, so you can obtain a reasonably correct answer rapidly.

4.1. EVALUATING ANSWERS

Whenever you perform a calculation, you should check the answer to be sure it is reasonable.

1. Is the answer physically possible? For example, a substance cannot have a negative weight or length. After a substance is heated, its temperature must be higher, not lower, than at the start.
2. Is the answer mathematically possible? If you have multiplied by a fraction, make sure that the size of the answer is appropriate—that is, larger than the original number if the fraction was greater than 1, smaller if the fraction was less than 1 (see Section 2.6). Perform an approximate calculation to make sure you have not made a gross mathematical error.
3. Are the units of the answer appropriate? Consider the units as if they were factors, and multiply or divide. If you are trying to calcu-

late the volume of a substance and you get an answer with units of $1/g^2$, the setup for the problem must have been wrong.

4. Is the answer the one called for in the problem? Be sure you have not left the calculation incomplete or calculated the wrong quantity.

PROBLEMS

4.1 Tell whether each answer is possible, and tell why or why not.
- (a) A sample of 200 g of water at a temperature of 100°C was added to a flask containing 100 g of water at a temperature of 50°C. The temperature of the mixture was calculated to be 120°C.
- (b) The calculation in part (a) was repeated. This time the answer was found to be 25°C.

4.2 Tell whether each of the reported results is possible and tell why or why not.
- (a) A compound of iron and sulfur was prepared, using 3.6 g of iron and 2.4 g of sulfur. The product weighed 7.5 g.
- (b) A mixture of 3.6 g of iron and 3.6 g of sulfur was heated. The resulting compound of iron and sulfur weighed 5.6 g.
- *(c) A piece of magnesium weighing 1.20 g was heated in air, and the magnesium oxide formed weighed 0.20 g.

4.3 Tell whether the result of each calculation is possible, and tell why or why not.
- *(a) A student calculated the mass of a substance by multiplying the volume, 37.9 cm^3, by the density, 4.2 g/cm^3.
- (b) A student calculated the volume of a liquid by multiplying the mass, 10.2 g, by the density, 0.80 g/cm^3.
- (c) A student converting 20.7 ounces to pounds multiplied by 16, getting an answer of 331 lb.
- (d) A student calculated that 25.00 mL of a solution of acetic acid in water contained 765 g of acetic acid.

4.2. APPROXIMATE CALCULATIONS

It is often useful to do an approximate calculation. This can be helpful in evaluating an answer, since it gives you a quick idea of the general size of the answer. If you find that an answer should be about 200 and your

calculator reads 0.8, you know that your calculation was wrong in some way.

Sometimes you don't need an exact answer. You might want to know whether the three items you wish to buy would cost more money than you have available. If the total price of the items would be around \$50 and you have \$30, you do not need to know the exact total to know that you cannot afford them. If you are performing an experiment in the laboratory that requires approximately 200 mL of solution and there is only 100 mL of solution available, you do not need to know whether you need exactly 180 mL or exactly 230 mL to know that a refill is needed.

For any rapid mental calculation, whether for an approximate answer or for an exact answer, the trick is to convert the problem to a series of operations that can be carried out easily.

The two general methods of simplifying problems are (1) round numbers off to ones that are easy to use; (2) cancel numbers that are approximately equal to each other, if one appears in the numerator and one in the denominator of a fraction. (See Section 2.3.C.)

■ EXAMPLE 1

What is the approximate price of three items that cost \$7.98 each?

Round off the price to \$8. Then $3 \times \$8 = \$24$. The actual price is *less* than that used in the calculation. Therefore, the actual total is *less* than \$24. ■

■ EXAMPLE 2

What is the approximate cost of a basket of groceries, if you are buying four items that cost \$1.99, two that cost 49¢, six that cost 39¢, and one that costs 89¢?

The easiest way to do such a calculation is to round everything off to whole dollars. Count as \$1 any two items costing about half a dollar or any three items costing approximately 33¢.

$$\begin{array}{lll}
4\ @\ \$1.99 = 4 \times \$2 & = \$8 \\
2\ @\ \ 0.49 & = \$1 \\
6\ @\ \ 0.39 = 2\,(3)(0.39) & = \$2 \\
1\ @\ \ 0.89 & = \underline{\$1} \\
\quad \text{Approximate total} & \ \ \$12 \text{ (actual total \$12.17)}
\end{array}$$

■

■ EXAMPLE 3

How much solution should you mix to do measurements on eight samples if each requires 210 mL? You will make up somewhat more

than the minimum required in case of waste, spills, and so forth. Therefore, you do not need to know the precise amount needed but must be sure you are making enough.

Rounding off, approximately 200 mL per sample is needed.

$$8 \text{ samples} \times 200 \text{ mL/sample} = 1600 \text{ mL}$$

The number used in calculation, 200, was less than the amount actually needed, 210, so the amount calculated is too small. The error is approximately 5%:

$$\frac{10 \text{ mL error}}{210 \text{ mL needed}} \times 100\% \approx 5\%$$

An additional 100 mL will more than make up for the 5% error and, allowing 100 mL more for waste, you should make up at least 1800 mL of solution. (In practice, you might well decide to make 2000 mL and have plenty.)

An alternative calculation might allow for waste on each sample and calculate that approximately 250 mL (to get a convenient number) should be prepared for each.

$$8 \text{ samples} \times 250 \text{ mL/sample} = 8 \times \frac{1000}{4} = 2000 \text{ mL}$$

This clearly is a little greater allowance for waste than in the first calculation, but it leads to approximately the same result. ■

In some problems you could round off to either a smaller or a larger number. Therefore, it is possible to minimize errors caused by rounding off. When two numbers are to be multiplied, if one is rounded down to a smaller number, try to round the other up to a larger number.

PROBLEMS

4.4 Calculate the approximate cost for each.

(a) Four items @ $9.95 (b) Eight cans @ $0.89

(c) Five items @ $2.50

(d) One package costing $1.98, three that each cost $0.45, and five that each cost $0.53

4.5 Calculate the approximate amount of material you need for each. Although you do not want to waste chemicals, you want to have a

little extra available in case it is needed.

(a) You need enough solution to use for four samples, each requiring 40–50 mL, and three samples, each requiring 25–40 mL.

(b) You obtain enough of a solid unknown to run a minimum of three samples. You may have to run up to three more, if results on the first three are not satisfactory. Each sample requires about 0.9 g.

There are some combinations of numbers that have approximately equal values and can therefore be canceled in an approximate calculation. For example, 9×11 is very close to 10×10. Notice that where 9 is 1 less than 10, 11 is 1 more than 10. Similarly, 8×12 is also close to 10×10 and, of course, to 9×11.

EXAMPLE 4

Find the approximate answer.

$$\frac{8 \times 6 \times 93}{(7)^2}$$

Since 8×6 is close to 7×7, they can be canceled.

$$\frac{\cancel{8 \times 6} \times 93}{\cancel{7 \times 7}} \approx 93$$

EXAMPLE 5

Multiply 2.7×38.

The usual way to round off would make both numbers larger and the resulting answer too large.

$$2.7 \times 38 \approx 3 \times 40 = 120$$

Rounding 2.7 all the way down to 2 would be such a large percentage change that again a large error would be introduced. However, 2.7 could be rounded down to 2.5, which in this calculation is a convenient number to use.

$$2.7 \times 38 \approx 2.5 \times 40 = 100$$

For comparison, the correct answer is 103. Therefore, the second method gave a more nearly correct answer. (Remember that we are not here attempting to find a precisely accurate answer but are instead using a rapid method to find an approximate answer.)

In a calculation involving *division*, it is desirable to round off in the same direction in both numerator and denominator. That is, if the numerator is rounded to a larger number, the denominator should also be rounded to a larger number. This is especially important if the rounding off introduces a large percentage change.

EXAMPLE 6

Find the approximate quotients: (a) 367/750, (b) 429/850.

In each fraction, the denominator might equally well be rounded up or down. For (a) it would be convenient to round 367 *up* to 400. Therefore, the choice should be to round the denominator up also.

$$\frac{367}{750} \approx \frac{400}{800} = 0.5$$

If the denominator had been rounded down to 700, the result would have been 0.57. Since the correct answer is 0.489, the procedure of rounding numerator and denominator in the same direction gave a more nearly correct answer.

For (b) the numerator will be rounded *down*, from 429 to 400. Therefore, the denominator should also be rounded down.

$$\frac{429}{850} \approx \frac{400}{800} = 0.5 \quad \text{The correct answer is 0.505.}$$ ■

Sometimes it is desirable to know not only whether the approximate answer is too big or too small, but also by how much. Then it is useful to calculate the percent error introduced by an approximation. (Calculating an approximate percent error is sufficient, or you might find yourself spending more time calculating the percent error than you saved by making the rapid approximation.)

EXAMPLE 7

Calculate the percent error for each: (a) 14 is rounded down to 10; (b) 1004 is rounded down to 1000. For each, the error is 4.

(a) $\%\ \text{error} = \frac{4}{14} \times 100\% \approx 30\%$

(b) $\%\ \text{error} = \frac{4}{1004} \times 100\% \approx 0.4\%$

Note that the same size change can result in very different percent errors. ■

■ EXAMPLE 8

Find the approximate answer:

$$\frac{2.0\,(450)}{0.082\,(298)}$$

There are several possible approaches to solving this problem; outlined next are examples of the approaches that might be used.

METHOD 1: Round 298 to 300. Notice that

$$2.0 \times 450 = 900$$

Then

$$\frac{\overset{3}{\cancel{900}}}{0.082 \times \cancel{300}} = \frac{3}{0.082}$$

To simplify the division, you might round 0.082 to 0.1, an increase of about 25%:

$$\frac{3}{0.1} = 30$$

METHOD 2: Round the 450 and 0.082 both up.

$$\frac{2.0\,(450)}{0.082\,(298)} \approx \frac{2.0\,(500)}{0.1\,(300)} = \frac{100\cancel{0}}{3\cancel{0}} = 33$$

METHOD 3: Rewrite in exponential notation with some rounding off.

$$\begin{aligned}\frac{2.0\,(450)}{0.082\,(298)} &\approx \frac{2.0 \times 4.5 \times 10^2}{8 \times 10^{-2} \times 3 \times 10^2} \\ &= \frac{\overset{3}{\cancel{9}}}{8 \times \cancel{3}} \times \frac{10^2}{10^{-2} \times 10^2} \\ &= \frac{3}{8} \times 10^2 \\ &= 0.38 \times 10^2 = 38\end{aligned}$$

The actual answer is 37. ■

The types of approximate calculation in Examples 6 and 8 are a useful check on the result obtained from a calculator. With practice, an approximate calculation can be made very rapidly, and the result will usually indicate whether the answer obtained on the calculator is reasonable.

PROBLEMS

4.6 Tell whether the result of each calculation is possible, and tell why or why not. Do not perform the calculations. [***Hint:*** See Section 2.6.]

*(a) $\dfrac{273}{298} \times 500 = 550$ (b) $\dfrac{298}{273} \times 200 = 218$

(c) $\dfrac{770}{760} \times 527 = 520$ (d) $\dfrac{770 \times 527}{62.4 \times 500} = 3.25 \times 10^{6}$

(e) $\dfrac{770 \times 527}{62.4 \times 500} = 1.30$

4.7 For each, do an approximate mental calculation and select the answer(s) that might be correct.

(a) $\dfrac{24}{46} \times 15 =$

(1) 0.35 (2) 7.8 (3) 35 (4) 78

(5) 1.7×10^{-3}

(b) $\dfrac{96}{2.0 \times 28} =$

(1) 0.053 (2) 1.7 (3) 1344 (4) 5376

4.8 Make rapid approximate calculations for each.

[***Hint:*** Write in exponential notation.]

(a) $\dfrac{27 \times 720}{59}$ (b) $\dfrac{250 \times 189}{93}$

*(c) $\dfrac{760 \times 22.4 \times 330}{900 \times 273}$ (d) $\dfrac{(9)^2 \times 620}{99 \times 0.80}$

(e) $\dfrac{3.5 \times 10^{-2}\,(750)}{0.082\,(310)}$

4.3. SIMPLIFIED ARITHMETIC

If you need an exact answer, not just an approximate one, you can often convert a calculation to one that is easier to carry out by using the rules of mathematics.

One way to simplify multiplication is to make use of the general rule

$$a\,(b + c) = ab + ac$$

Essentially, this is the method used in multiplication of numbers with more than one digit.

$$\begin{aligned} 7 \times 13 &= 7\,(10 + 3) \\ &= 7\,(10) + 7\,(3) \\ &= 70 + 21 = 91 \end{aligned}$$

Of course, c may equally well be subtracted.

$$a\,(b - c) = ab - ac$$

To use this method to calculate the total price of three items that cost \$7.98 each, rewrite the price as \$8.00 – \$0.02. Then

$$\begin{aligned} 3\,(\$7.98) &= 3\,(\$8.00 - \$0.02) \\ &= 3\,(\$8.00) - 3\,(\$0.02) \\ &= \$24.00 - \$0.06 \\ &= \$23.94 \end{aligned}$$

How would you know that you should rewrite \$7.98 as \$8.00 – \$0.02? Look for the closest convenient number. "Convenient number" usually means one with only one digit that is not 0, but this may depend on your own preference. If you find it easy to multiply by 11, then there is no advantage to regrouping 11 as 10 + 1. On the other hand, many people have trouble multiplying by 9 or 8, even though these are one-digit numbers. If you have this difficulty, you might want to think of 9 as 10 – 1 or of 8 as 10 – 2.

$$\begin{aligned} 9 \times 7 &= (10 - 1)(7) \\ &= 70 - 7 = 63 \end{aligned}$$

$$\begin{aligned} 8 \times 6 &= (10 - 2)(6) \\ &= 60 - 12 = 48 \end{aligned}$$

There may be a choice of convenient methods. For instance, 26 is 20 + 6, but it is also 30 – 4 or 25 + 1. Since the reason for using these methods is to make the calculation easy for you, pick the one *you* prefer.

PROBLEM

4.9 Express as sums or differences of numbers easily multiplied. For some, three terms may be useful.

(a) 692	(b) 309	(c) \$2.95	(d) \$10.98
(e) 89	(f) 77	(g) 23	(h) 46
(i) 18	(j) \$1.89		

There are many other possibilities for conversion of an arithmetic problem into one that can be done quickly in your head. Since multiplication and division may be done in any order, you can choose a convenient sequence.

$$\begin{aligned} 40 \times 22 &= (4 \times 10) \times 22 \\ &= (4 \times 22) \times 10 \\ &= 88 \times 10 = 880 \end{aligned}$$

By leaving the 10 until last, you have less to keep in mind for the multiplication.

A less obvious, but very useful, simplification can be made when you are required to multiply by 50 or 25 (or by the same numbers with the decimal moved, such as 5, 0.5, 2.5, 0.25, etc.). Remember that 50 is 100/2 and 25 is 100/4.

EXAMPLE 9

Multiply 88 by 50.

$$\begin{aligned} 88 \times 50 &= 88 \times \frac{100}{2} \\ &= \frac{88}{2} \times 100 \\ &= 44 \times 100 = 4400 \end{aligned}$$

■

EXAMPLE 10

Multiply 88 by 2.5.

$$\begin{aligned} 88 \times 2.5 &= 88 \times \frac{10}{4} \\ &= \frac{88}{4} \times 10 \\ &= 22 \times 10 = 220 \end{aligned}$$ ■

■ EXAMPLE 11

Divide 320 by 25.

$$\begin{aligned} \frac{320}{25} &= \frac{320}{\frac{100}{4}} \\ &= 320 \times \frac{4}{100} \\ &= \frac{320}{100} \times 4 \\ &= 3.2 \times 4 = 12.8 \end{aligned}$$ ■

When one number is to be divided by another, rounding off can sometimes be done in such a way that one number is especially easy to divide by another.

■ EXAMPLE 12

$$\frac{298}{2 \times 15} \approx \frac{300}{30} = 10$$

$$\frac{685}{702} \approx \frac{700}{700} = 1$$ The actual quotient should be less than 1, since the numerator is less than the denominator. ■

In some calculations there may be several numbers to be canceled in order to reach a setup that allows a rapid mental calculation.

■ EXAMPLE 13

What is the approximate answer?

$$\frac{273 \times 775 \times 350}{298 \times 760}$$

You can make a very gross approximation and consider $273 \approx 298$ and $775 \approx 760$, so that they cancel.

$$\frac{273}{298} \approx 1 \qquad \frac{777}{760} \approx 1$$

$$\frac{\cancel{273}}{\cancel{298}} \times \frac{\cancel{775}}{\cancel{760}} \times 350 \approx 1 \times 1 \times 350 = 350$$

Alternatively, you might notice that the numbers 273 and 298 are close to 270 and 300 and that these are divisible by 30.

$$\frac{273}{298} \approx \frac{270}{300} = \frac{9}{10} = 0.9 = 1 - 0.1$$

$$\overset{0.9}{\frac{273}{298}} \times \overset{1}{\frac{\cancel{775}}{\cancel{760}}} \times 350 \approx (1 - 0.1)(1)(350) = 350 - 35 = 315$$

The actual answer turns out to be 327, so both methods of approximation give very reasonable results. ■

PROBLEMS

4.10 Perform the multiplication indicated mentally. (Simplify the operation as before; then multiply and add or subtract.)

*(a) 89×3 (b) $\$1.05 \times 9$
(c) $\$1.98 \times 7$ (d) $\$3.95 \times 8$
(e) 72×30 (f) 44×60

4.11 Perform the multiplication or division mentally.

*(a) 36×5.0 (b) $\dfrac{440}{50}$
(c) 28×25 (d) $\dfrac{120}{25}$
(e) 120×0.25 (f) 69×33.3

SOLUTIONS TO STARRED PROBLEMS

4.2(c) No. Draw a piece of metal. Add something (oxygen) to it to form magnesium oxide. You would predict that the mass of the product would be the sum of the masses of the magnesium and the added oxygen, so 0.20 g would be much too small.

4.3(a) Yes. The units for the answer are

$$\text{cm}^3 \times \frac{\text{g}}{\text{cm}^3} = \text{g} \qquad \text{This is an appropriate unit of mass.}$$

4.6(a) No. Multiplication by a fraction that has the numerator smaller than the denominator should give a product that is smaller than the original number.

4.8(c) Although there is nothing that looks easy to cancel, you are not trying to obtain an exact answer. As a rapid approximation,

$$\frac{760}{900} \sim 1 \qquad \text{and} \qquad \frac{330}{273} \sim 1$$

so

$$\frac{760}{900} \times \frac{330}{273} \times 22.4 \approx 1 \times 1 \times 22.4 \approx 22.4$$

The actual answer is 22.9, so this is within 2% of the correct answer. The fact that one fraction had a value of less than 1 and the other had a value of more than 1 meant that the outrageous approximations counteracted each other to some extent.

4.10(a) To multiply 89 by 3, think of 89 as 90 − 1.

$$\begin{aligned} 89\ (3) &= (90 - 1)(3) \\ &= 90\ (3) - 1\ (3) \\ &= 270 - 3 = 267 \end{aligned}$$

4.11(a) Consider 50 as 100/2. Then divide by 2 and multiply by 100.

$$36 \times 50 = 36 \times \frac{100}{2} = \frac{36}{2} \times 100 = 1800$$

CHAPTER

5

SETTING UP PROBLEMS: DIMENSIONAL ANALYSIS

5.1. DIMENSIONAL ANALYSIS

A great variety of problems can be solved by using an approach called *dimensional analysis* or the *unit-factor method*. This method makes use of the units or dimensions of the quantities involved as a guide in setting up a calculation.

A quantity of a given size can be described in different units, and the numerical value will be different for these different units. A given distance could be called 2 yd or 6 ft or 72 in.; the distance does not change even though the units and their related numbers change. Similarly, you have the same purchasing power with 1 dollar bill, 100 pennies, 4 quarters, 10 dimes, or 20 nickels. Again the amount of money is the same, but the number of coins needed will depend on the size of the coin. A great many calculations require expressing a quantity in different but related units; you must calculate the numerical value that goes with those units.

To change a quantity from one set of units to another, the original quantity must be multiplied by a conversion factor (see Tables 1.1 and 1.3). This factor must not change the size of the quantity even though it changes the numbers and the units. How can this be done? The size of a quantity is not changed if it is multiplied by 1; [illegible]

TABLE 5.1 CONVERSION FACTORS*

60 seconds =	1 minute
60 minutes =	1 hour
†454 grams =	1 pound (lb)
16 ounces =	1 pound
†2.54 cm =	1 inch
†1.06 quart =	1 L
32 fluid ounces =	1 L
4 quarts =	1 gallon
12 inches =	1 foot
3 feet =	1 yard
5280 feet =	1 mile

* The exact units, such as 1 lb, are exactly 1, with as many significant figures as needed.

† These units represent measurements. They have been rounded off to three significant figures.

The procedure for solving a problem by dimensional analysis involves the following steps:

1. Write the units of the answer needed.
2. Set this equal to the quantity given, including units, times some conversion factor(s).
3. Multiply the original quantity given by whatever factors are needed to convert its units to the units needed. Each factor will be a fraction in which the numerator is equivalent to the denominator.

How does one choose conversion factors for step 3? Units can be canceled in the same way as numbers. Hence the units must be set up in such a way that the units of the starting quantity cancel out, leaving only the units needed for the answer. A unit appearing in the numerator can be canceled if the same unit appears in the denominator. (It does not matter whether the word used for the unit is singular or plural; "feet" and "foot" are the same measuring unit and can be canceled. However, "square feet" is a unit of area and is not equal to "feet," a unit of length.) This use of units to determine whether to multiply or divide is the source of the great power of dimensional analysis in solving problems. Let us look at some specific examples.

EXAMPLE 1

How many yards are there in 6 ft?

STEP 1: The unit of the answer must be yards.

STEP 2: The quantity given is in feet. Therefore,

$$\text{yards} = 6 \text{ ft} \times \underline{\qquad}$$

STEP 3: The factor used must have "feet" in the denominator to cancel the unit given (remember that whole numbers can be considered to be numerators of a fraction of which the denominator is 1) and "yards" in the numerator to introduce the unit needed. Since the numerator must be equivalent to the denominator, the number of yards in the numerator must equal the number of feet in the denominator. The relationship between yards and feet is known.

$$1 \text{ yd} = 3 \text{ ft}$$

Therefore,

$$\begin{aligned} \text{yards} &= 6 \cancel{\text{ft}} \times \frac{1 \text{ yd}}{3 \cancel{\text{ft}}} \\ &= 2 \text{ yd} \end{aligned}$$

Note that the factor required in Example 1 had yards divided by feet.

If the problem had been set up incorrectly,

$$\textit{Wrong:} \quad \begin{aligned} \text{yards} &= 6 \text{ ft} \times \frac{3 \text{ ft}}{1 \text{ yd}} \\ &= 18 \frac{\text{ft}^2}{\text{yd}} \end{aligned}$$

the answer would have had the absurd units, feet squared per yard. A drawing would also show that the resulting numbers were wrong. Yards are large, and it does not make sense that there should be many big yards equal to a small number of smaller feet. ■

■ EXAMPLE 2

How many liters are there in 250 mL?

STEP 1: The units needed are liters.

STEP 2: The units given are milliliters. Therefore,

$$\text{L} = 250 \text{ mL} \times \underline{\qquad}$$

STEP 3: The unit needed must relate milliliters and liters and have liters in the numerator and milliliters in the denominator. We know 1 mL =

10^{-3} L. Therefore, the factor needed is 10^{-3} L/1 mL. The equation is

$$\mathrm{L} = 250\ \cancel{\mathrm{mL}} \times \frac{10^{-3}\ \mathrm{L}}{1\ \cancel{\mathrm{mL}}}$$

$$= 0.250\ \mathrm{L} = 2.50 \times 10^{-2}\ \mathrm{L}$$

■

EXAMPLE 3

A rectangular object measures 50.0 cm by 150 cm. What is the area of the object in square meters (m^2)?

The area is found by

$$A = 50.0\ \mathrm{cm} \times 150\ \mathrm{cm} = 7500\ \mathrm{cm}^2$$

STEP 1: The unit of the answer must be m^2.
STEP 2: The unit of the given data is cm^2.

$$\mathrm{m}^2 = 7500\ \mathrm{cm}^2 \times \underline{\qquad}$$

STEP 3: The factor needed must have cm^2 in the denominator and m^2 in the numerator, but there is no such factor given in Table 1.1. There is, however, a factor relating m to cm, and using this as a factor twice (squaring it) will give the factor needed.

$$\mathrm{m}^2 = 7500\ \mathrm{cm}^2 \times \frac{10^{-2}\ \mathrm{m}}{1\ \mathrm{cm}} \times \frac{10^{-2}\ \mathrm{m}}{1\ \mathrm{cm}}$$

$$= 7.50 \times 10^3\ \cancel{\mathrm{cm}^2} \times \frac{10^{-4}\ \mathrm{m}^2}{1\ \cancel{\mathrm{cm}^2}} = 7.50 \times 10^{-1}\ \mathrm{m}^2$$

Since the measurements in the original problem had three significant figures, the answer must have three. ■

In Example 3 a conversion factor had to be used twice to produce the units needed. Very often, two or more different conversion factors must be used.

EXAMPLE 4

How many grams are there in 2.0 oz (ounces)?
STEP 1: The unit needed is g.

STEP 2: The unit given is oz, so

$$g = 2.0 \text{ oz} \times \underline{\qquad}$$

STEP 3: The factor needed must have g in the numerator and ounces in the denominator. Here we encounter a difficulty. Table 5.1 gives a relationship between g and lb but not between g and ounces. We can solve the problem by using two factors. The first will convert the unit given, oz, to lb.

$$16 \text{ oz} = 1 \text{ lb}$$

Then we are ready to convert the new unit, lb, to g.
PRELIMINARY STEP: Convert the unit given, 2.0 oz, to lb.

$$\text{lb} = 2.0 \cancel{\text{oz}} \times \frac{1 \text{ lb}}{16 \cancel{\text{oz}}}$$

$$= \frac{2.0}{16} \text{ lb}$$

Then, using the relationship 454 g = 1 lb, we find that

$$g = \frac{2.0}{16} \cancel{\text{lb}} \times \frac{454 \text{ g}}{1 \cancel{\text{lb}}}$$

$$= 57 \text{ g}$$

Note that since the data were given only to two significant figures, there is no justification for giving more in the answer.

The two steps could have been combined:

$$g = 2.0 \cancel{\text{oz}} \times \frac{1 \cancel{\text{lb}}}{16 \cancel{\text{oz}}} \times \frac{454 \text{ g}}{1 \cancel{\text{lb}}}$$

$$= 57 \text{ g}$$

■

How can you tell whether to use more than one factor? If you know a relationship between the units you are given and those you need, you can do the calculation with one factor. If you do not know a direct relationship between the units given and those needed, you must use more than one factor. Sometimes three or more factors must be used. For ratio units, such as miles per hour, you may need some factors to convert the units of the numerator and others to convert the units of the denominator. The relationships you "know" depend on the table of

equivalences you are using. These can all be developed from a few definitions, however, and it is often faster and simpler to do a calculation with two or three factors than to spend the time finding a table that lists the exact relationship you need.

EXAMPLE 5

If you are driving at 60 miles/hr, how many feet do you travel in 1 sec?
STEP 1: The unit needed is ft in every sec, ft/sec.
STEP 2: The unit given is miles in every hr, miles/hr.
STEP 3: Two conversions must be done; ft to miles and hr to sec. For the latter, note that hr must be in the numerator in order to cancel and that sec are needed in the denominator. We do not know a direct relationship between hr and sec, but we can convert hr to min and then min to sec.

Either the conversion factor for time or that for distance may be written first.

$$\frac{\text{ft}}{\text{sec}} = 60\,\frac{\cancel{\text{miles}}}{\text{hr}} \times \frac{5280\ \text{ft}}{1\ \cancel{\text{mile}}} \times \underline{\qquad}$$

Now that miles have been converted to feet, the conversion from hours to seconds will be done in two steps. It is helpful, where several factors must be used, to write down the units resulting from each multiplication. There is no advantage to doing the arithmetic step by step, and sometimes a disadvantage, since numbers appearing in the various factors may turn out to cancel conveniently.

$$\frac{\text{ft}}{\text{sec}} = 60\,\frac{\cancel{\text{miles}}}{\cancel{\text{hr}}} \times \frac{5280\ \text{ft}}{1\ \cancel{\text{mile}}} \times \frac{1\ \cancel{\text{hr}}}{60\ \cancel{\text{min}}} \times \frac{1\ \cancel{\text{min}}}{60\ \text{sec}}$$

$$\text{(resulting units)}\ \left.\frac{\text{ft}}{\text{hr}}\right| \qquad \left.\frac{\text{ft}}{\text{min}}\right| \qquad \left.\frac{\text{ft}}{\text{sec}}\right|$$

$$= 88\,\frac{\text{ft}}{\text{sec}}$$

■

EXAMPLE 6

You are traveling at 60 miles/hr. How many cm are you traveling in 1 sec? To solve this problem, you can start out as in Example 4, adding the conversion from ft to cm. However, if you have the result of Example 4, you may use it here. That is, knowing that 60 miles/hr = 88 ft/sec, you can treat the problem as one of converting from 88 ft/sec to cm/sec.
STEP 1: The unit needed is cm/sec.

STEP 2: Given is 88 ft/sec.

$$\frac{\text{cm}}{\text{sec}} = 88\,\frac{\text{ft}}{\text{sec}} \times \underline{\qquad}$$

STEP 3: There is no need to change the time unit. We know the relationship between cm and in. from the table and can convert ft to in.:

$$1 \text{ ft} = 12 \text{ in.}$$

$$1 \text{ in.} = 2.54 \text{ cm}$$

Therefore,

$$\frac{\text{cm}}{\text{sec}} = \frac{88\,\cancel{\text{ft}}}{\text{sec}} \times \frac{12\,\cancel{\text{in.}}}{1\,\cancel{\text{ft}}} \times \frac{2.54 \text{ cm}}{1\,\cancel{\text{in.}}}$$

$$= 2700\,\frac{\text{cm}}{\text{sec}} = 2.7 \times 10^3\,\frac{\text{cm}}{\text{sec}}$$ ■

PROBLEMS

5.1 Perform the unit conversions indicated.
- *(a) 29 g to kg
- (b) 208 mL to L
- (c) 0.153 kg to g
- (d) 1.5 m to cm
- (e) 20.0 lb to g
- (f) 3.0 liters to qt
- (g) 3.0 qt to liters
- (h) 5/8 in. to cm

5.2 Perform the unit conversions indicated.
- (a) 0.153 kg to mg
- (b) 20.0 lb to kg
- (c) 2.00 yd to cm
- (d) How many grams of aspirin are contained in a 5-grain aspirin tablet? 1 grain = 0.0023 oz.

5.3 In American measures, 1 qt = 4 cups = 8 fluid ounces. One cup = 16 tablespoons = 48 teaspoons. Calculate the number of mL in each of the following. Use your answer for part (a) to work the other parts.
- *(a) 1.00 cup
- (b) 1.0 tablespoon
- (c) 1.0 teaspoon
- (d) 1.0 jigger (2.0 oz)

5.4 U.S. track meets may be run using English units of distance, but international meets use metric units. Calculate the distance in meters that corresponds to each distance listed.
(a) 100 yards (b) 440 yards *(c) 1.00 mile

5.5 Calculate the distance in yards that corresponds to each distance listed.
(a) 100 meters (b) 1000 meters (c) 5000 meters

5.6 Distance runners often participate in "6K" or "10K" runs. Calculate the distance in miles corresponding to each:
(a) 6.00 km (b) 10.0 km

5.7 A swimming pool is 40 feet long, and you swim the length of the pool 40 times (40 laps). How far did you swim in
(a) miles (b) meters

5.8 (a) In 1988, Italy imposed a highway speed limit of 110 km/hr. What is this in miles/hour? [*Hint:* See answer to 5.4(c).]
(b) The speed limit on most American highways is 55 miles/hr. What is this in km/hour?
(c) If you are traveling 35 miles/hr, how many feet do you travel in one second?

5.9 The speed of light is 3.00×10^{10} cm/sec.
*(a) How fast is this in miles/sec? Use this answer for parts (b) and (c).
(b) How fast is this in miles/hr?
(c) The moon is about 240,000 miles from the earth. How many seconds would it take a radio signal from earth traveling at the speed of light to reach the moon?

5.2. SOLVING CHEMISTRY PROBLEMS BY DIMENSIONAL ANALYSIS

There are many chemistry problems that are easily solved by dimensional analysis. A great many definitions are expressions of relationships between different units for a quantity.

EXAMPLE 7

The definition of density is the ratio of mass (incorrectly called "weight") to volume. In metric units, the density is given as mass in grams of 1 mL or of 1 cm^3 (these are the same, since 1 mL = 1 cm^3). If the density of a liquid is 0.80 g/mL, what is the mass of 25 mL of the liquid?

STEP 1: The quantity needed is mass; the unit of mass is g.
STEP 2: The quantity given is 25 mL. Therefore,

$$g = 25 \text{ mL} \times \underline{\qquad}$$

STEP 3: The relationship between g and mL is given by the density

$$0.80 \text{ g} = 1 \text{ mL}$$

Then

$$g = 25 \cancel{\text{mL}} \times \frac{0.80 \text{ g}}{1 \cancel{\text{mL}}}$$

$$= 20 \text{ g}$$

■

■ EXAMPLE 8

A metal has a density of 3.1 g/cm^3. What is the volume of a piece of the metal that weighs 80 g?
STEP 1: The quantity needed is volume; the unit is cm^3.
STEP 2: The quantity given is 80 g. Therefore,

$$cm^3 = 80 \text{ g} \times \underline{\qquad}$$

STEP 3: The relationship between g and cm^3 for the metal is given.

$$3.1 \text{ g} = 1 \text{ cm}^3$$

Then

$$\text{mL} = 80 \cancel{g} \times \frac{1 \text{ cm}^3}{3.1 \cancel{g}}$$

$$= 26 \text{ cm}^3$$

(Be sure to write the factor in such a way that the units cancel properly.)

■

PROBLEMS

5.10 Ethyl alcohol has a density of 0.80 g/mL.
 (a) How many g in 50 mL of ethyl alcohol?
 *(b) How many mL of ethyl alcohol are needed to obtain 1.6 g?
 (c) How many mL in 24 g of ethyl alcohol?

5.11 Seawater has a density of 1.025 g/mL.

*(a) How many grams are there in 1.000 Liter of seawater?
(b) How many grams are there in 1.000 qt of seawater?
(c) What is the volume in mL of 1.000 kg of seawater?
(d) What is the volume in mL of 1.00 kg of pure water, density 1.00 g/mL?

5.3. MOLES

The very important chemical unit, the *mole*, is defined in several ways. One mole of any substance is the amount of the substance that contains 6.02×10^{23} particles. One mole is the amount of a substance that has a mass equal to the formula weight of the substance in grams. That is, if a molecule has a formula weight of 60 amu, 1 mole of such molecules has a mass of 60 g. Another way of saying this is that the formula weight is the number of grams in 1 mole.

The formula weight is computed by adding the atomic weights of all the atoms present in one unit or particle of the substance, whether that unit or particle is an atom, a molecule, or a pair of ions. Therefore, the useful relationships are

$$1 \text{ mole} = 6.02 \times 10^{23} \text{ particles}$$

1 mole has a mass of M grams, where M is the formula weight

There are a great many calculations in chemistry that make use of the mole, many of them coming under the general category of "stoichiometry" or calculations of the quantities of chemical reactions. These calculations are so common and so fundamental that it is worthwhile to practice until you can do them rapidly and confidently. Most such problems are easily solved by use of dimensional analysis.

EXAMPLE 9

How many moles of S are present in 64 g of S?
UNIT NEEDED: moles of S.
QUANTITY GIVEN: 64 g. Therefore,

$$\text{moles of S} = 64 \text{ g S} \times \underline{\qquad}$$

The relationship between g and moles is

1 mole S atoms weighs 32 g (from a table of atomic weights)

Therefore,

$$\text{moles of S} = 64 \text{ g of S} \times \frac{1 \text{ mole of S}}{32 \text{ g of S}}$$

$$= 2.0 \text{ moles of S}$$

■

■ EXAMPLE 10

How many S atoms are there in 2.5 moles of sulfur, S?
UNIT NEEDED: S atoms.
UNIT GIVEN: 2.5 moles of S. Therefore,

$$\text{S atoms} = 2.5 \text{ moles of S} \times \underline{\qquad}$$

The relationship between moles and atoms is

$$1 \text{ mole} = 6.0 \times 10^{23} \text{ atoms}$$

Then

$$\text{S atoms} = 2.5 \cancel{\text{moles}} \text{ of S} \times \frac{6.0 \times 10^{23} \text{ S atoms}}{1 \cancel{\text{mole}} \text{ of S}}$$

$$= 15 \times 10^{23} \text{ S atoms} = 1.5 \times 10^{24} \text{ S atoms}$$

■

■ EXAMPLE 11

A sample of copper, Cu, contains 3.0×10^{22} atoms. How many moles and how many grams of copper are present?

First, find how many moles. The given unit is atoms.

$$\text{moles of Cu} = 3.0 \times 10^{22} \text{ Cu atoms} \times \underline{\qquad}$$

The relationship between moles and atoms is

$$1 \text{ mole} = 6.0 \times 10^{23} \text{ atoms}$$

Therefore,

$$\text{moles of Cu} = 3.0 \times 10^{22} \text{ Cu } \cancel{\text{atoms}} \times \frac{1 \text{ mole of Cu}}{6.0 \times 10^{23} \text{ Cu } \cancel{\text{atoms}}}$$

$$= 5.0 \times 10^{-2} \text{ mole of Cu} = 0.050 \text{ mole of Cu}$$

Now find the number of grams present, using the number of moles from the last calculation as the given unit.

$$\text{g of Cu} = 5.0 \times 10^{-2} \text{ mole of Cu} \times \underline{\qquad}$$

The relationship between g and moles is the atomic weight of copper,

$$1 \text{ mole of Cu} = 64 \text{ g of Cu}$$

Then

$$\text{g of Cu} = 5.0 \times 10^{-2} \cancel{\text{mole}} \text{ of Cu} \times \frac{64 \text{ g of Cu}}{1 \cancel{\text{mole}} \text{ of Cu}}$$

$$= 3.2 \text{ g of Cu}$$

EXAMPLE 12

How many molecules are there in 13.2 g of CO_2? The molecular weight of CO_2 is 44 (12 for C, 16 for each of 2 O's).
UNIT NEEDED: molecules of CO_2.
UNIT GIVEN: g

$$\text{molecules of } CO_2 = 13.2 \text{ g of } CO_2 \times \underline{\qquad}$$

No direct relationship between g and molecules is known. However, in Example 9 we went from g to moles, and in Example 10 we went from moles to molecules. Combining these two steps, with care to make the units cancel correctly, gives the entire setup.
RELATIONSHIPS:

$$1 \text{ mole} = 6.0 \times 10^{23} \text{ molecules}$$

$$1 \text{ mole of } CO_2 = 44 \text{ g of } CO_2 \text{ (the molecular weight)}$$

The entire setup is

$$\text{molecules of } CO_2 = 13.2 \cancel{\text{g}}\, CO_2 \times \frac{1 \cancel{\text{mole}}\, CO_2}{44 \cancel{\text{g}}\, CO_2} \times \frac{6.0 \times 10^{23} \text{ molecules of } CO_2}{1 \cancel{\text{mole}} \text{ of } CO_2}$$

$$= 1.8 \times 10^{23} \text{ molecules of } CO_2$$

Note that one must sometimes multiply by the molecular weight and sometimes divide. Similarly, one must sometimes multiply and some-

times divide by the number of atoms or molecules in a mole. It is important to see that the factors are set up to make the units cancel correctly. It is also useful to check the answer to see if it makes sense. A fraction of an atom, or 10^{-23} atom, is impossible, although one may well have a fraction of a mole. A mole is a very large number of individual particles, and one may have many or few of such particles. Since each mole weighs 1 g or more, there cannot be many moles for a few g.

EXAMPLE 13

In a "vacuum" there may be 10^{-7} mole of air in 1 L. How many molecules is that?

UNIT NEEDED: molecules.

UNIT GIVEN: 10^{-7} mole of air. (The statements about the vacuum and the volume of 1 L being considered simply define the problem and are not part of the calculation.)

$$\text{molecules} = 10^{-7}\ \text{mole} \times \underline{\qquad}$$

RELATIONSHIP:

$$1\ \text{mole} = 6 \times 10^{23}\ \text{molecules}$$

$$\text{molecules} = 10^{-7}\ \cancel{\text{mole}} \times \frac{6 \times 10^{23}\ \text{molecules}}{1\ \cancel{\text{mole}}}$$

$$= 6 \times 10^{16}\ \text{molecules}$$

(Think about this answer. Is the vacuum *really* empty?) ■

PROBLEMS

5.12 How many moles in each?

*(a) 64 g of O_2 (molecular weight 32 g = 1 mole)
(b) 64 g of Cu (64 g = 1 mole)
(c) 64 g of H_2SO_4 (98 g = 1 mole)
(d) 0.0070 g of N_2 (28 g = 1 mole)
(e) 0.21 g of C_2H_6 (30 g = 1 mole)
*(f) 1.000 lb of sugar ($C_{12}H_{22}O_{11}$) (342 g = 1 mole)
(g) 1.000 kg of iron, Fe, (1 mole = 55.8 g)

5.13 How many moles in each?

*(a) 6×10^{22} molecules
(b) 1.2×10^{24} molecules
(c) 3×10^{23} molecules

5.14 How many g in each?
*(a) 3.0 moles of O_2 (32 g = 1 mole)
(b) 10.0 moles of Fe (56 g = 1 mole)
(c) 0.0010 mole of CO_2 (44 g = 1 mole)
*(d) 3.0×10^{23} molecules of CO_2
(e) 3.0×10^{24} molecules of O_2
(f) 9.0×10^{21} molecules of O_2

5.4. CONCENTRATION

An important group of ratio units is that which expresses the concentration of a solution. There are several such units, expressing the ratio of the amount of solute (dissolved substance) to the volume or mass of the solution (or, in one instance, the mass of solvent). The amount of solute may be expressed as moles, equivalents, or grams.

Two concentration units frequently encountered in chemistry are defined as follows:

$$\text{molarity, } M = \frac{\text{moles of solute}}{\text{liters of solution}}$$

$$\text{weight percent (or \%)} = \frac{\text{g solute}}{\text{100 g solution}}$$

$$= \frac{\text{g solute}}{\text{g solution}} \times 100\%$$

Each definition can be used in either of two ways. You can use the definition as a formula, substitute the known quantities, and solve the equation for the remaining quantity. Alternatively, you can use the definition as a conversion factor, as was done with density.

When the concentration of a solution is used as a conversion factor, the specified quantity of solute is treated as if it were equivalent to the specified quantity of solution. That is, since a 2 M solution has 2 moles of solute in 1 liter of solution, you would use the relationship

$$\text{2 moles solute} = \text{1 liter of solution}$$

as the conversion factor for your calculation.

EXAMPLE 14

How many moles of HCl are in 2 L of 3 M HCl solution?
UNIT NEEDED: moles of HCl.
GIVEN: 2 liters of solution.

Relationship: The solution contains 3 moles of HCl for every 1 liter of solution.

$$\text{moles HCl} = 2 \cancel{\text{L sol'n}} \times \frac{3 \text{ moles HCl}}{1 \cancel{\text{L sol'n}}}$$
$$= 6 \text{ moles HCl}$$

(The term "sol'n" is a convenient abbreviation of "solution.") ■

■ EXAMPLE 15

How many liters of 2.0 *M* NaOH solution are needed to obtain 0.50 mole of NaOH?
Needed: liters of solution.
Given: 0.50 mole of NaOH.
Relationship: 2.0 moles of NaOH in every 1 L of solution.

$$\text{liters} = 0.50 \cancel{\text{mole NaOH}} \times \frac{1 \text{ L sol'n}}{2.0 \cancel{\text{moles NaOH}}}$$
$$= 0.25 \text{ L}$$
■

■ EXAMPLE 16

How many g of glucose are in 500 g of 2% glucose solution?
Needed: g of glucose.
Given: 500 g of solution.
Relationship: 2 g of glucose to every 100 g of solution.

$$\text{g glucose} = 500 \text{ g sol'n} \times \frac{2 \text{ g glucose}}{100 \text{ g sol'n}}$$
$$= 10 \text{ g}$$
■

PROBLEMS

5.15 How many moles of HCl are there in each of the following?
- (a) 3 L of 2 *M* solution
- (b) 3.0 L of 6.0 *M* solution
- *(c) 250 mL of 2 *M* solution
- (d) 100 mL of 1 *M* solution

(e) 100 mL of 12 *M* solution
(f) 10 mL of 10^{-2} *M* solution

5.16 How many liters and how many mL of 3.0 *M* H_2SO_4 solution are needed to get the quantity specified?
*(a) 0.50 mole of H_2SO_4
(b) 6 moles of H_2SO_4
(c) 3×10^{-2} mole of H_2SO_4
(d) 1.0 mole of H_2SO_4
*(e) 98 g of H_2SO_4 (98 g/mole)
[***Hint:*** How many moles is that?]
(f) 32.7 g of H_2SO_4
(g) 32.7 mg of H_2SO_4

5.17 You have a 2.0% solution of NaCl.
*(a) How many g of NaCl in 250 g of solution?
(b) How many g of NaCl in 50 g of solution?
(c) How many g of NaCl in 10 g of solution?
(d) How many g of solution are needed to get 3.0 g of NaCl?
(e) How many g of solution are needed to get 10 g of NaCl?
*(f) How many g of solution are needed to get 1.0 mole of NaCl (58.5 g/mole)?

5.5. USING BALANCED EQUATIONS

The last few pages have shown how problems could be solved by using definitions as conversion factors and using the units as a guide to setting up the calculation. There are other problems that can be solved using as conversion factors quantities that are chemically equivalent in some way, even though it would be nonsense to call them mathematically equal.

Balanced equations for chemical reactions tell what is put in (the reactants) and what is formed (the products) and in what proportions. The substances are shown by their chemical formulas; each formula gives the number of atoms of each type in the substance, as shown in the subscripts following the symbols for the atoms. Thus the formula H_2O indicates a molecule made up of two atoms of H to one of O, with the subscript 1 omitted. The number of molecules of each type is shown as a coefficient written before the formula, so 3 H_2O means that three molecules are used, each having the formula H_2O. As with subscripts, a coefficient of 1 is not written. These coefficients show the *ratio* of molecules of each type used and formed rather than the total. The coefficients can equally well be read as showing the ratio of moles used in the reaction. This is true because the number of molecules in a mole is the same for all substances.

To make use of a balanced equation in a calculation, consider the relative amounts used as chemically equivalent. In the equation

$$2H_2 + O_2 = 2H_2O$$

the coefficients state that the proportions used are two molecules of H_2 to every one molecule of O_2 to every two molecules of H_2O formed. Equally, the ratio could be stated

$$2 \text{ moles } H_2 \text{ to } 1 \text{ mole } O_2 \text{ to } 2 \text{ moles } H_2O$$

In setting up a problem, the ratio

$$\frac{2 \text{ moles } H_2}{1 \text{ mole } O_2}$$

can be used as a conversion factor, since the two quantities are chemically equivalent.

EXAMPLE 17

How many moles of H_2O are formed in the preceding reaction if 6.0 moles of O_2 are used? How many g of H_2O is that? The molecular weight of H_2O is 18 g/mole.
NEEDED: moles of H_2O formed.
GIVEN: 6.0 moles of O_2 used.
RELATIONSHIP: From the balanced equation, 2 moles of H_2O are formed for every 1 mole of O_2 used. Therefore,

$$\text{moles } H_2O = 6.0 \cancel{\text{ moles } O_2} \times \frac{2 \text{ moles } H_2O}{1 \cancel{\text{ mole } O_2}}$$

$$= 12 \text{ moles } H_2O$$

To find the number of grams,
GIVEN: 12 moles of H_2O formed.
NEEDED: g of H_2O formed.
RELATIONSHIP: 1 mole of H_2O = 18 g (the molecular weight).

$$g\ H_2O = 12 \cancel{\text{ moles } H_2O} \times \frac{18 \text{ g } H_2O}{1 \cancel{\text{ mole } H_2O}}$$

$$= 220 \text{ g}$$

It is important to recognize that **all calculations using coefficients from balanced equations must be done in terms of moles or molecules.** If the original quantity is given in some other unit, such as grams, **the first step in the calculation is to find the number of moles.** So many chemical calculations are done in this way that a good rule of thumb is to start each problem by asking: How many moles?

EXAMPLE 18

The equation for the commercial production of ammonia, NH_3, is

$$N_2 + 3H_2 = 2NH_3$$

How many moles and how many g of NH_3 can be produced from 24 g of H_2 and sufficient N_2 to react with it?
MOLECULAR WEIGHTS: N_2, 28; H_2, 2.0; NH_3, 17.

Since the equation gives the relationship in moles, the first thing to do is to convert grams of H_2 to moles of H_2. This can be done in a separate step, or the steps can be combined, as was done in the unit conversions where more than one conversion factor must be used.
NEEDED: moles of NH_3.
GIVEN: 24 g of H_2.
RELATIONSHIPS: 1 mole of H_2 = 2.0 g of H_2

3 moles of H_2 will form 2 moles of NH_3

(The 3 and 2 are the coefficients in the equation.)
SETUP:

$$\text{moles } NH_3 = 24\ \cancel{\text{g } H_2} \times \frac{1\ \cancel{\text{mole } H_2}}{2.0\ \cancel{\text{g } H_2}} \times \frac{2 \text{ moles } NH_3}{3\ \cancel{\text{moles } H_2}}$$

$$= 8.0 \text{ moles } NH_3$$

Then the number of grams of NH_3 can be calculated.
NEEDED: g of NH_3.
GIVEN: 8.0 moles of NH_3.
RELATIONSHIP: 1 mole of NH_3 = 17 g NH_3

$$\text{g } NH_3 = 8.0\ \cancel{\text{moles } NH_3} \times \frac{17 \text{ g } NH_3}{1\ \cancel{\text{mole } NH_3}}$$

$$= 140 \text{ g } NH_3$$

■

PROBLEMS

5.18 The equation for the fermentation of sugar, $C_{12}H_{22}O_{11}$, to alcohol, C_2H_6O, is

$$H_2O + C_{12}H_{22}O_{11} = 4C_2H_6O + 4CO_2$$

Molecular weights: $C_{12}H_{22}O_{11}$, 342; C_2H_6O, 46; CO_2, 44.

(a) Five pounds of sugar is about 6.6 moles. How many moles of alcohol are formed in the fermentation of 6.6 moles of sugar? How many moles of CO_2?

*(b) If 72 g of sugar is used, how many moles and how many g of alcohol are formed?

(c) How many moles and how many g of CO_2 are formed in part (b)?

(d) Under certain conditions 1 mole of a gas has a volume of 22.4 liters. What is the volume of the CO_2 gas formed in part (a) under these conditions? What is the volume formed in part (c)?

5.19 The equation for a reaction is

$$3C_3H_8O + K_2Cr_2O_7 + 4H_2SO_4$$
$$= Cr_2(SO_4)_3 + 7H_2O + 3C_3H_6O + K_2SO_4$$

(a) If 1.5 moles of C_3H_8O are used, how many moles of $K_2Cr_2O_7$ and of H_2SO_4 are needed?

(b) How many g of C_3H_6O (molecular weight 58) are formed from 1.5 moles of C_3H_8O?

(c) How many g of C_3H_6O are formed from 18 g of C_3H_8O (molecular weight 60)?

*(d) How many moles of H_2SO_4 are used in the reaction with 18 g of C_3H_8O? How many mL of 18 *M* solution would provide this many moles?

(e) How many g of $K_2Cr_2O_7$ (formula weight 294) are needed to make 29 g of C_3H_6O?

SOLUTIONS TO STARRED PROBLEMS

5.1(a) Step 1: Unit needed: kg.
Step 2: Quantity given: 29 g.

$$kg = 29\ g \times \underline{\qquad}$$

(e) 100 mL of 12 *M* solution
(f) 10 mL of 10^{-2} *M* solution

5.16 How many liters and how many mL of 3.0 *M* H_2SO_4 solution are needed to get the quantity specified?
*(a) 0.50 mole of H_2SO_4
(b) 6 moles of H_2SO_4
(c) 3×10^{-2} mole of H_2SO_4
(d) 1.0 mole of H_2SO_4
*(e) 98 g of H_2SO_4 (98 g/mole)
[*Hint:* How many moles is that?]
(f) 32.7 g of H_2SO_4
(g) 32.7 mg of H_2SO_4

5.17 You have a 2.0% solution of NaCl.
*(a) How many g of NaCl in 250 g of solution?
(b) How many g of NaCl in 50 g of solution?
(c) How many g of NaCl in 10 g of solution?
(d) How many g of solution are needed to get 3.0 g of NaCl?
(e) How many g of solution are needed to get 10 g of NaCl?
*(f) How many g of solution are needed to get 1.0 mole of NaCl (58.5 g/mole)?

5.5. USING BALANCED EQUATIONS

The last few pages have shown how problems could be solved by using definitions as conversion factors and using the units as a guide to setting up the calculation. There are other problems that can be solved using as conversion factors quantities that are chemically equivalent in some way, even though it would be nonsense to call them mathematically equal.

Balanced equations for chemical reactions tell what is put in (the reactants) and what is formed (the products) and in what proportions. The substances are shown by their chemical formulas; each formula gives the number of atoms of each type in the substance, as shown in the subscripts following the symbols for the atoms. Thus the formula H_2O indicates a molecule made up of two atoms of H to one of O, with the subscript 1 omitted. The number of molecules of each type is shown as a coefficient written before the formula, so 3 H_2O means that three molecules are used, each having the formula H_2O. As with subscripts, a coefficient of 1 is not written. These coefficients show the *ratio* of molecules of each type used and formed rather than the total. The coefficients can equally well be read as showing the ratio of moles used in the reaction. This is true because the number of molecules in a mole is the same for all substances.

To make use of a balanced equation in a calculation, consider the relative amounts used as chemically equivalent. In the equation

$$2H_2 + O_2 = 2H_2O$$

the coefficients state that the proportions used are two molecules of H_2 to every one molecule of O_2 to every two molecules of H_2O formed. Equally, the ratio could be stated

$$2 \text{ moles } H_2 \text{ to } 1 \text{ mole } O_2 \text{ to } 2 \text{ moles } H_2O$$

In setting up a problem, the ratio

$$\frac{2 \text{ moles } H_2}{1 \text{ mole } O_2}$$

can be used as a conversion factor, since the two quantities are chemically equivalent.

■ EXAMPLE 17

How many moles of H_2O are formed in the preceding reaction if 6.0 moles of O_2 are used? How many g of H_2O is that? The molecular weight of H_2O is 18 g/mole.
NEEDED: moles of H_2O formed.
GIVEN: 6.0 moles of O_2 used.
RELATIONSHIP: From the balanced equation, 2 moles of H_2O are formed for every 1 mole of O_2 used. Therefore,

$$\text{moles } H_2O = 6.0 \cancel{\text{moles } O_2} \times \frac{2 \text{ moles } H_2O}{1 \cancel{\text{mole } O_2}}$$

$$= 12 \text{ moles } H_2O$$

To find the number of grams,
GIVEN: 12 moles of H_2O formed.
NEEDED: g of H_2O formed.
RELATIONSHIP: 1 mole of H_2O = 18 g (the molecular weight).

$$g\ H_2O = 12 \cancel{\text{moles } H_2O} \times \frac{18 \text{ g } H_2O}{1 \cancel{\text{mole } H_2O}}$$

$$= 220 \text{ g}$$

■

RELATIONSHIP: $1 \text{ kg} = 10^3 \text{ g}$.
STEP 3:

$$\text{kg} = 29 \text{ g} \times \frac{1 \text{ kg}}{10^3 \text{ g}}$$
$$= 0.029 \text{ kg}$$

The conversion factor is written with kg in the numerator, where it will be needed, and g in the denominator to cancel out the g in the original quantity.

5.3(a) STEP 1: Unit needed: mL.
STEP 2: Quantity given: 1.00 cup.

$$\text{mL} = 1.00 \text{ cup} \times \underline{\qquad}$$

RELATIONSHIP: Several factors will be needed.

$$1 \text{ qt} = 4 \text{ cups} \qquad \text{(given in the problem)}$$
$$1.06 \text{ qt} = 1 \text{ L} \qquad \text{(from Table 5.1)}$$
$$1 \text{ mL} = 10^{-3} \text{ L}$$

STEP 3:

$$\begin{array}{lcccc} \text{mL} = 1.00 \text{ cup} & \times \dfrac{1 \text{ qt}}{4 \text{ cups}} & \times \dfrac{1 \text{ L}}{1.06 \text{ qt}} & \times \dfrac{1 \text{ mL}}{10^{-3} \text{ L}} \\ \text{(result)} & \text{qt}| & \text{L}| & \text{mL}| \end{array}$$
$$= 236 \text{ mL}$$

First, cups are converted to qt. Then qt are converted to L. Finally, liters are converted to mL.

5.4(c)

$$\text{m} = 1 \text{ mile} \times \frac{5280 \text{ ft}}{1 \text{ mile}} \times \frac{12 \text{ in.}}{1 \text{ ft}} \times \frac{2.54 \text{ cm}}{1 \text{ in.}} \times \frac{10^{-2} \text{ m}}{1 \text{ cm}}$$
$$= 1.61 \times 10^3 \text{ m}$$

5.9(a) STEP 1: The unit needed is miles/sec.
STEP 2: The quantity given is 3.00×10^{16} cm/sec.

$$\frac{\text{miles}}{\text{sec}} = 3.00 \times 10^{10} \frac{\text{cm}}{\text{sec}} \times \underline{\qquad}$$

The unit sec needs no conversion, but several factors are needed to go from cm to miles. The unit in the table is

$$2.54 \text{ cm} = 1 \text{ in.}$$

Familiar conversions can be used to get from in. to miles.

$$12 \text{ in.} = 1 \text{ ft}$$

$$5280 \text{ ft} = 1 \text{ mile}$$

STEP 3:

$$\frac{\text{miles}}{\text{sec}} = 3.00 \times 10^{10} \frac{\text{cm}}{\text{sec}} \times \frac{1 \text{ in.}}{2.54 \text{ cm}} \times \frac{1 \text{ ft}}{12 \text{ in.}} \times \frac{1 \text{ mile}}{5280 \text{ ft}}$$

$$\frac{\text{in.}}{\text{sec}} \qquad \frac{\text{ft}}{\text{sec}} \qquad \frac{\text{miles}}{\text{sec}}$$

$$= 1.86 \times 10^{5} \frac{\text{miles}}{\text{sec}}$$

The use of exponential notation is very helpful in checking the position of the decimal in such a calculation.

5.10(b) The statement of density gives the relationship between g and mL.
UNIT NEEDED: mL.
QUANTITY GIVEN: 1.6 g

$$\text{mL} = 1.6 \text{ g} \times \_\_\_\_\_$$

RELATIONSHIP: 0.80 g = 1 mL.

$$\text{mL} = 1.6 \text{ g} \times \frac{1 \text{ mL}}{0.80 \text{ g}} \quad \text{set up to cancel g}$$

$$= 2.0 \text{ mL}$$

5.11(a) NEEDED: g.
GIVEN: 1.000 L of seawater.
RELATIONSHIP: 1.025 g = 1 mL. To get to liters, 1 mL = 10^{-3} L.

$$\text{g} = 1.000 \text{ L} \times \frac{1 \text{ mL}}{10^{-3} \text{ L}} \times \frac{1.025 \text{ g}}{1 \text{ mL}}$$

$$= 1025 \text{ g}$$

5.12(a) NEEDED: moles.
GIVEN: 64 g of O_2.
RELATIONSHIP: 32 g = 1 mole.

$$\text{moles } O_2 = 64 \text{ g } O_2 \times \frac{1 \text{ mole } O_2}{32 \text{ g } O_2}$$
$$= 2.0 \text{ moles } O_2$$

(f) NEEDED: moles.
GIVEN: 1.000 lb of sugar.
Two conversion factors are needed, first to grams, then to moles.
RELATIONSHIPS:

$$454 \text{ g} = 1 \text{ lb}$$
$$342 \text{ g} = 1 \text{ mole}$$

$$\text{moles sugar} = 1.000 \text{ lb sugar} \times \frac{454 \text{ g}}{1 \text{ lb}} \times \frac{1 \text{ mole sugar}}{342 \text{ g sugar}}$$
$$= 1.33 \text{ moles}$$

5.13(a) NEEDED: moles.
GIVEN: 6×10^{22} molecules.
RELATIONSHIP: 1 mole = 6×10^{23} molecules.

$$\text{moles} = 6 \times 10^{22} \text{ molecules} \times \frac{1 \text{ mole}}{6 \times 10^{23} \text{ molecules}}$$
$$= 1 \times 10^{-1} \text{ mole}$$

5.14(a) NEEDED: g
GIVEN: 3.0 moles of O_2.
RELATIONSHIP: 32 g of O_2 = 1 mole of O_2.

$$\text{g of } O_2 = 3.0 \text{ moles } O_2 \times \frac{32 \text{ g } O_2}{1 \text{ mole } O_2}$$
$$= 96 \text{ g } O_2$$

(d) Two conversion factors are needed: first to moles, then to grams.

NEEDED: g of CO_2.
GIVEN: 3.0×10^{23} molecules of CO_2.
RELATIONSHIPS:

$$1 \text{ mole } CO_2 = 44 \text{ g } CO_2$$

$$1 \text{ mole} = 6.0 \times 10^{23} \text{ molecuies}$$

$$g = 3.0 \times 10^{23} \text{ molecules} \times \frac{1 \text{ mole}}{6.0 \times 10^{23} \text{ molecules}} \times \frac{44 \text{ g } CO_2}{1 \text{ mole}}$$

$$= 22 \text{ g of } CO_2$$

5.15(c) NEEDED: moles of HCl.
GIVEN: 250 mL of solution.
RELATIONSHIP: The solution is 2 *M*. This means that

$$1 \text{ L contains 2 moles of HCl}$$

Also:

$$1 \text{ mL} = 10^{-3} \text{ L}$$

$$\text{moles HCl} = 250 \text{ mL sol'n} \times \frac{10^{-3} \text{ L}}{1 \text{ mL}} \times \frac{2 \text{ moles HCl}}{1 \text{ L sol'n}}$$

$$= 0.5 \text{ mole HCl}$$

5.16(a) NEEDED: mL of solution.
GIVEN: 0.50 mole of H_2SO_4.
RELATIONSHIP: A 3.0 *M* solution is one that contains 3.0 moles of H_2SO_4 in 1 L of solution.

$$\text{L sol'n} = 0.50 \text{ mole } H_2SO_4 \times \frac{1 \text{ L sol'n}}{3 \text{ moles } H_2SO_4}$$

$$= 0.17 \text{ L sol'n}$$

$$\text{mL} = 0.17 \text{ L sol'n} \times \frac{1 \text{ mL}}{10^{-3} \text{ L}}$$

$$= 170 \text{ mL sol'n}$$

(e) NEEDED: liters of solution.
GIVEN: 98 g of H_2SO_4.

RELATIONSHIPS: g of H_2SO_4 must be converted to moles. One liter of solution contains 3.0 moles of H_2SO_4.

$$\text{L sol'n} = 98 \text{ g } H_2SO_4 \times \frac{1 \text{ mole}}{98 \text{ g}} \times \frac{1 \text{ L sol'n}}{3.0 \text{ moles } H_2SO_4}$$

$$= 0.33 \text{ L sol'n}$$

$$\text{mL sol'n} = 0.33 \text{ L sol'n} \times \frac{1 \text{ mL}}{10^{-3} \text{ L}}$$

$$= 330 \text{ mL sol'n}$$

5.17(a) NEEDED: g of NaCl.
GIVEN: 250 g of solution.
RELATIONSHIP: 2.0 g of NaCl in every 100 g of solution (from the definition of a 2.0% solution).

$$\text{g NaCl} = 250 \text{ g sol'n} \times \frac{2.0 \text{ g NaCl}}{100 \text{ g sol'n}}$$

$$= 5.0 \text{ g NaCl}$$

(f) One way to answer the question is to consider that the question asking for 1.0 mole of NaCl is in effect asking for the grams of solution that contain 58.5 g of NaCl.
SETUP:

$$\text{g sol'n} = 58.5 \text{ g NaCl} \times \frac{100 \text{ g sol'n}}{2.0 \text{ g NaCl}}$$

$$= 2900 \text{ g sol'n}$$

Alternatively, the setup could have used two conversion factors.

1 mole = 58.5 g NaCl

100 g sol'n contains 2.0 g NaCl

$$\text{g} = 1 \text{ mole NaCl} \times \frac{58.5 \text{ g NaCl}}{1 \text{ mol NaCl}} \times \frac{100 \text{ g sol'n}}{2.0 \text{ g NaCl}}$$

$$= 2900 \text{ g sol'n}$$

5.18(b) Before the equation can be used, the quantities must be in moles. Therefore, a conversion to moles is needed.
RELATIONSHIPS AVAILABLE:

1 mole sugar = 342 g

1 mole alcohol = 46 g

Four moles of alcohol are formed from 1 mole of sugar (coefficients from the equation).
NEEDED: moles of alcohol.
GIVEN: 72 g of sugar.

$$\text{moles alcohol} = 72 \text{ g sugar} \times \frac{1 \text{ mole sugar}}{342 \text{ g sugar}} \times \frac{4 \text{ moles alcohol}}{1 \text{ mole sugar}}$$

$$= 0.84 \text{ mole alcohol}$$

$$\text{grams alcohol} = 0.84 \text{ mole} \times \frac{46 \text{ g alcohol}}{1 \text{ mole}}$$

$$= 39 \text{ g alcohol}$$

5.19(d) The first step is to convert to moles of C_3H_8O so that the equation can be used. Then, using the equation, find moles of H_2SO_4 used. Finally, find how many mL of H_2SO_4 solution are used.
NEEDED: moles of H_2SO_4.
GIVEN: 18 g of C_3H_8O.
RELATIONSHIPS:

$$1 \text{ mole } C_3H_8O = 60 \text{ g [from part (c)]}$$

$$3 \text{ moles } C_3H_8O \text{ react with } 4 \text{ moles } H_2SO_4$$

$$\text{moles } H_2SO_4 = 18 \text{ g } C_3H_8O \times \frac{1 \text{ mole } C_3H_8O}{60 \text{ g}} \times \frac{4 \text{ moles } H_2SO_4}{3 \text{ moles } C_3H_8O}$$

$$= 0.40 \text{ mole } H_2SO_4$$

To find the mL of solution:
GIVEN: 0.40 mole of H_2SO_4.
RELATIONSHIPS:

$$1 \text{ L sol'n contains } 18 \text{ moles } H_2SO_4$$

$$1 \text{ mL} = 10^{-3} \text{ L}$$

SETUP:

$$\text{mL sol'n} = 0.40 \text{ mole } H_2SO_4 \times \frac{1 \text{ L sol'n}}{18 \text{ moles } H_2SO_4} \times \frac{1 \text{ mL}}{10^{-3} \text{ L}}$$

$$= 22 \text{ mL sol'n}$$

CHAPTER 6

GRAPHS

A special kind of drawing, a graph, is used very often in scientific work. What makes a graph different from other drawings is that it represents specific numerical values as well as a general pattern. That means that you can use a graph to determine, for instance, that the quantity of a substance present increased for the first 20 minutes of an experiment, then leveled off at 0.45 g, and didn't change any further. A newspaper may show two lines on the same graph to compare the change of cost of housing in an area with the average income of families over the last 10 years.

In scientific work, graphs are used to show general patterns, to represent experimental data in a way that is easy to read, to find numerical values of quantities, to help determine an equation that describes a phenomenon, and to solve equations.

6.1. CARTESIAN COORDINATES

The most common way in which data are plotted on a graph is in *Cartesian coordinates*. For these, two axes are drawn at right angles. One quantity is plotted on the horizontal axis (the abscissa) and the other on the vertical axis (the ordinate). If the numbers on the axes start at 0, the place where the axes cross is called the *origin*. (For convenience, often only a portion of such a graph may be shown. If the only interesting part is between the values of 100 and 120, there may be no need to show the whole graph all the way back to 0, and there would be the disadvantage of using space for what is not important.)

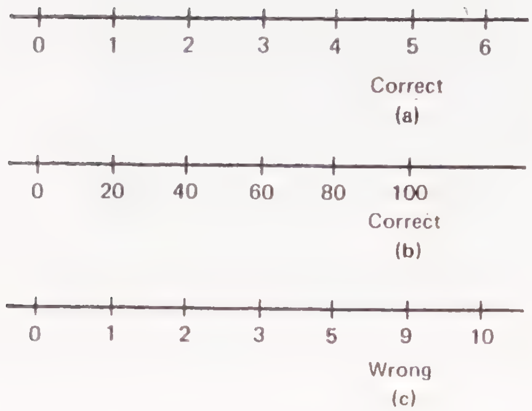


FIGURE 6.1

The abscissa is often called the x-axis, since the value of x is usually plotted along it, and the ordinate is the y-axis. There is no requirement that the quantities be symbolized by x and y. Normally, the independent variable is plotted on the x-axis and the dependent variable on the y-axis. For example, time is usually plotted on the horizontal axis.

Each quantity increases in value in proportion to its distance from the origin, positive values going to the right or up, and negative values going to the left or down. Since equal distances must represent equal numerical values, the values of the quantity must be evenly spaced along the axis. Thus the spacing in Figure 6.1(a) and (b) is correct, but that in 6.1(c) is not. Numbers are plotted on a graph according to where they fall within this even spacing. A value of 1.5 would then be plotted (marked) halfway

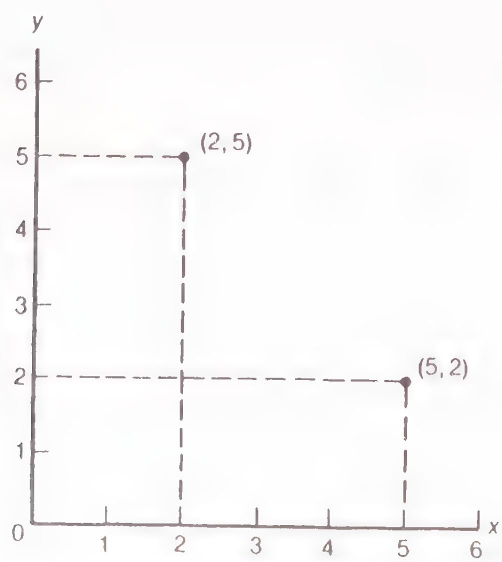


FIGURE 6 2

between the 1 and the 2. Even if the values measured were 1, 2, 4, 5, 9, and 10, they would still be plotted on a graph marked like that shown in Figure 6.1(a), never like that shown in Figure 6.1(c).

To define a point on a graph, the values of both quantities are specified, the one on the horizontal axis always being given first. Thus the point where $x = 2$ and $y = 5$, with x shown on the horizontal axis, is called the point (2, 5). Such an *ordered pair* of numerals can be given for any point. It is called an *ordered pair* because the order, or sequence, of the numbers is a part of the information; the point (5, 2) is not the same as the point (2, 5); see Figure 6.2. To plot a point on a graph, label the axes. Then measure off the correct number of spaces to the right to show the x value. Draw a vertical line (either actually or mentally) and measure up this line far enough to show the y value. Mark the point clearly.

6.2. GRAPHING EXPERIMENTAL DATA

When you wish to plot experimental data on a graph, you must make a number of decisions. The first is what kind of paper to use. There may be a choice of which quantity should go on which axis. There will certainly be a choice of how to space the numbers on the axis. Since experimental measurements always have some sources of error associated with the measurement, the true value may be slightly larger or smaller than the measured value, and the curve you draw can go directly through the point or a little above or below it. Which should it be? If the data do not give a straight line, could you get a straight line if you used the values of x^2 or $1/x$ or $\log x$, rather than x?

Students sometimes try to draw graphs on notebook paper or on paper with wide-spaced markings. This is a waste of time; the points cannot be placed accurately, nor can values be read accurately. "Quarter-inch cross-section" paper, with lines one-fourth inch apart, is seldom useful for graphing data. The ideal graph paper has close-spaced lines, with heavier lines every fifth or tenth line to make counting easy. For scientific work, the most useful paper has lines 1 mm apart, with heavier lines every 1 cm (10 mm).

Give the graph a title to show what it represents. Label the coordinates to show what is plotted on each and the units of the quantities. Place the independent variable on the abscissa (the horizontal axis) and the dependent variable on the ordinate (the vertical axis). In an experiment it is usually easy to tell which is independent; that is the one that is changed, and the change in the other is a result. When a quantity is measured over a period of time, time is obviously independent. The passage of time is not a result of a change but quite an independent process. In an experiment frequently done in beginning chemistry classes,

measured quantities of sulfur are added to pieces of copper, and the students find how much sulfur combined with the copper compared to the amount put in. Here the choice was of the amount of sulfur to add, that is, the independent quantity. The result is the amount combined, which is therefore the dependent quantity.

The spacing along the coordinates depends on the range to be graphed. You must look at the range of numbers and divide the space evenly to accommodate the full range (see Figure 6.3). It is not necessary to start at 0 for every graph. If the quantities range from 300 to 450, the space from 0 to 300 would just be wasted. To make room for the wasted space, the part of the graph between 300 and 450 would have to be compressed into only one third of the total space. It is desirable to spread a graph over a relatively large space, since that makes it possible to locate points more precisely. Whatever the choice of range to be covered, the space *must* be divided evenly so that a given distance on the graph is directly proportional to a given range of measurement.

Once the coordinates have been labeled appropriately, the experimental measurements are plotted. As always, measure over the correct distance for the first number in the pair, then up the correct distance for the second. Mark each point clearly.

Sometimes, instead of drawing a dot for a measured point, scientists draw a bar or a circle. These indicate the possible range of values for the measurement, including experimental error. For example, a 50-mL burette can be read to the nearest 0.01 mL, but the last figure is estimated and uncertain by about 0.02 mL. Therefore, a reading of 5.73 mL is really between 5.72 and 5.74 mL. This error is inherent in the measurement and does not represent a mistake by the experimenter. For very precise graphing, a line might be drawn from 5.72 to 5.74 rather than a point at 5.73.

When all the points have been placed on the graph and checked to see that they were placed correctly, the points are connected by a smooth curve. Beginning students sometimes make the mistake of trying to connect all the points, rather like a child's "follow-the-dot" drawing or like Figure 6.4(a). Experimental data contain both the kinds of errors that result from the limitations of the measuring instruments and other, harder-to-predict errors.

FIGURE 6.3 Two ways to label a coordinate to be used for graphing data in the range 300 to 450.

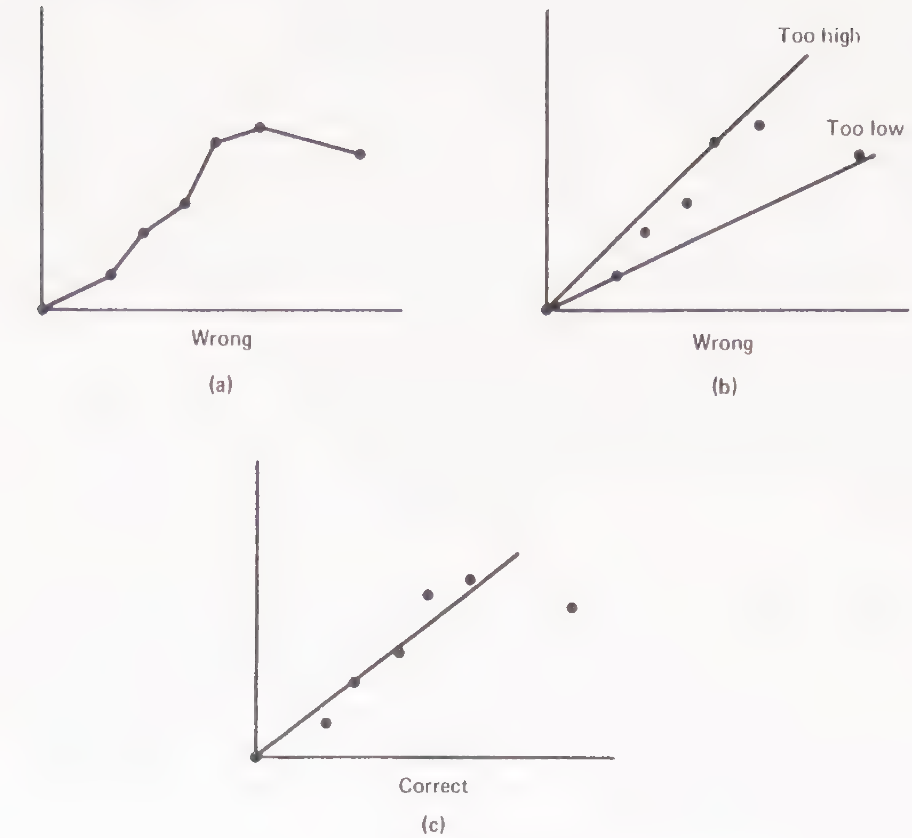


FIGURE 6.4

Since errors, as well as uncertainties in reading instruments, are usually random (as likely to result in high as in low measurements), the aim in graphing the data is to average out the errors, to draw the graph so that some points are above it and some below (Figure 6.4). The graph should be close to as many points as possible.

Occasionally, one point is so far from the rest that it does not seem to belong on the same line. The decision about what to do with that point depends very much on the circumstances and may not be clear without a lot more experiments. If there is good reason to believe that the point is erroneous, it may be ignored. A measurement is not judged to be erroneous just because it does not fit a theory. Sometimes the experimenter is aware of some difficulty or discrepancy in one measurement. "I wondered if I read the balance correctly that time." "That sample did not look like the rest; I suspect I didn't mix it enough." Other times, such as when comparing a series of compounds, there may be reason to believe that one compound does not behave like the others in a series. In any case the measurement should be explained, not ignored, in describing the experiment, even if it is ignored in drawing the graph.

A different situation occurs where one measurement is separated by

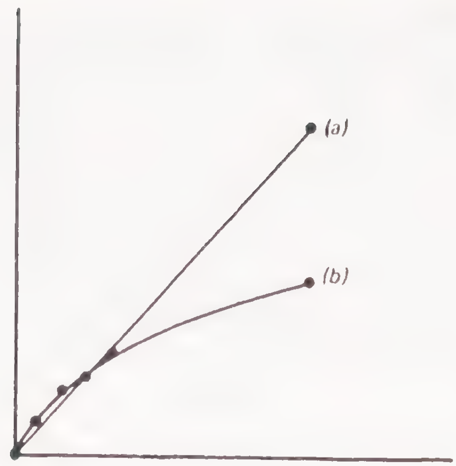


FIGURE 6.5

quite a distance from the others. For instance, if measurements are made at 1, 2, 3, 4, and 60 min, the point at 60 min should tell quite a lot about the correct shape of the curve and thus about the behavior observed. In Figure 6.5 the decision on the shape of the curve depends on which point, *a* or *b*, is the one measured. It would be useful, in case of doubt, to do another experiment making measurements at points in between those previously measured.

PROBLEMS

6.1 Draw the graphs of the data indicated. Plot x or time on the horizontal axis.

(a)

x	0	1	2	3	4
y	0	2	4	6	8

(b)

x	0	1	2	3	4	5
y	10	9	8	7	6	5

(c)

x	0	1	4	9	16
y	0	2	4	6	8

(d)

v	50	100	150	300	600
Time (sec)	10	20	30	60	120

(e)

d	1	5	8	15
Time (min)	2	10	15	30

6.2 Consider the following experimental data for measurements on a gas sample. Draw graphs as specified, being careful to label the axes clearly and to use the entire space. Comment on the information conveyed by the shape of each graph.

P (Torr)	800	760	700	650	600	500	450	400
V (mL)	235	250	270	290	320	380	420	480

(a) Plot P versus V. (A graph of P versus V is a graph having P on the vertical axis and V on the horizontal axis.)
(b) Calculate 1/V for each value of V. Plot P versus 1/V. [*Hint:* When each number on an axis has the same exponential part, you can include the exponential as part of the definition of the unit plotted and mark only the coefficients along the axis.]
(c) Calculate the product PV for each measurement. Plot PV versus P.

6.3. READING GRAPHS

Plotting points on a grid is not especially useful in itself. When the points are connected to give a smooth curve (called a "curve" even when it is a straight line), the graph becomes useful in a number of ways.

The graph serves as a visual "picture" of the data. It can show whether measurements represent a trend of some sort, or whether each event is unrelated to the others. If there is a trend, the graph lets you see what it is.

There are two ways to read a graph. One is to read specific values from the graph, as shown on Figure 6.13. The other is to look for the overall picture represented by the graph. The picture may be simple or complicated, but it can be easier to grasp than the same data presented as a table of numbers.

To understand the picture presented by a graph, you need to remem-

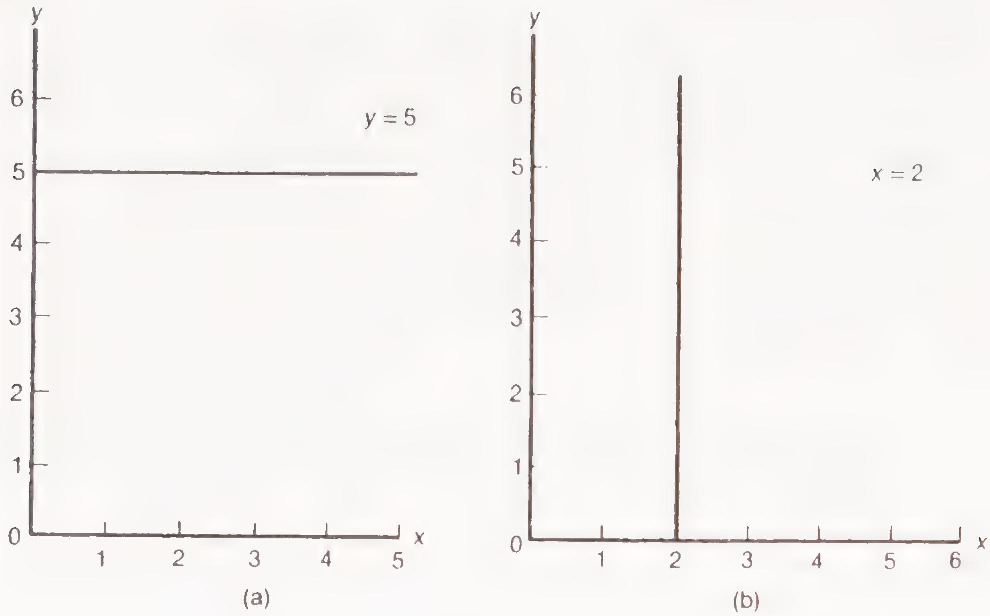


FIGURE 6.6

ber that the size of the quantity shown on the vertical axis increases as you go up (and decreases as you go down) and the size of the quantity shown on the horizontal axis increases as you move from left to right. A horizontal line, Figure 6.6(a), shows a situation in which y is the same no matter what the value of x. Here, $y = 5$. A vertical line, Figure 6.6(b), shows a situation in which x is the same no matter what the value of y. Here, $x = 2$.

Most real situations involve changes in both quantities at once. Graphs like Figure 6.7(a) and 6.7(b) show the concentration of a com-

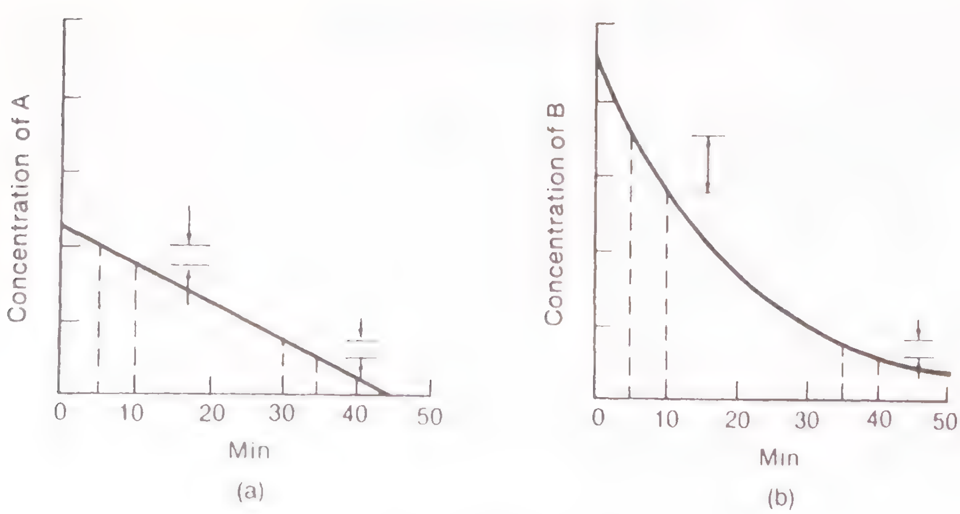


FIGURE 6.7

pound, plotted on the vertical axis, decreasing with increasing time, shown on the horizontal axis. This is what happens as a substance is used up in a chemical reaction. The fact that the shapes of the two graphs are different shows that there are several differences between the reactions.

1. The reaction of substance A goes at a steady rate until it is used up. Mark off intervals on this graph and see that the changes are the same in the different intervals. The reaction of substance B slows down as time goes by. This is shown by the fact that the change in the size of the concentration of B is smaller for a given time interval at the right of the graph than it is at the left. The dashed lines mark off equal intervals on the horizontal axis. Look at how much the curve goes down in each of these regions. The change in one quantity that accompanies a given change in the other is called the *slope* of the curve. This is discussed in Section 6.5.
2. At first, substance B reacts faster than does substance A. (The decrease in concentration of B is greater for a given time interval than is the decrease in the concentration of A. Later on, substance B is reacting slower than substance A.)
3. From the different behavior of A and B, one can deduce that the equations describing the rates of the two reactions must be different.

Two different things may be shown on the same graph. Figure 6.8 shows the changes in the concentrations of two substances. It shows that A is being used up as time goes on (the concentration of A decreases going to the right) and that B is being formed (the concentration of B is increasing).

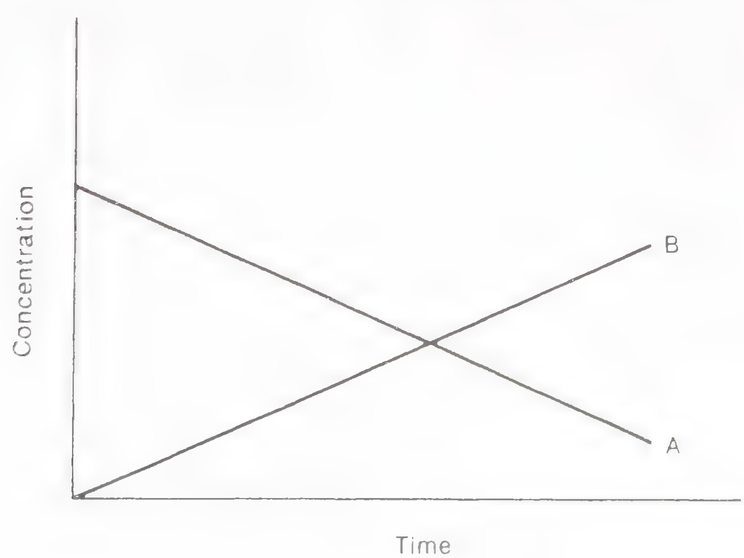


FIGURE 6.8

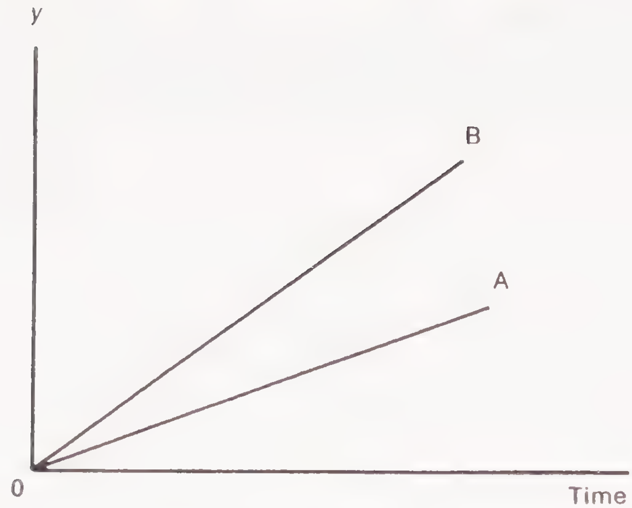


FIGURE 6.9

Figure 6.9 shows two unrelated processes on the same graph. By plotting them on the same graph, you can see easily that B is increasing faster than A.

The graph in Figure 6.10 shows a situation in which the behavior of a substance is different at different temperatures.

A pure liquid is cooled at a steady rate and the temperature measured from time to time. The temperature is plotted against time. This graph shows at first just what one would expect. During the time the liquid is cooled (going to the right), the temperature goes down. Then, however,

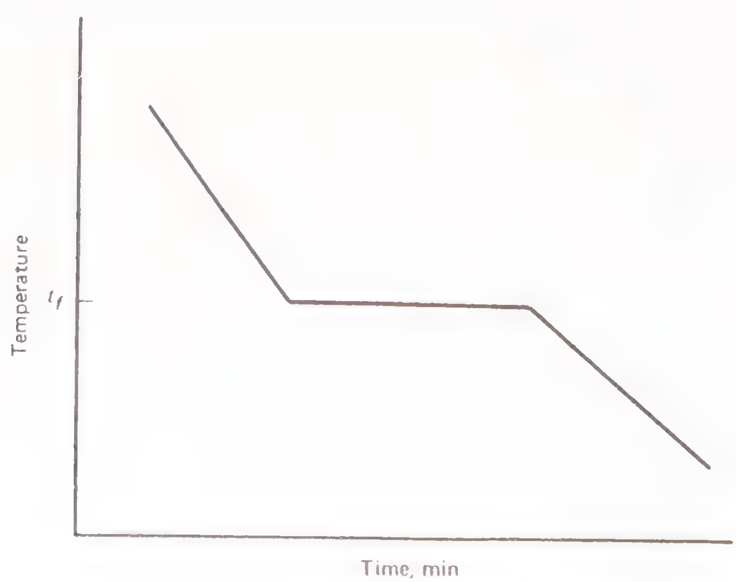


FIGURE 6.10 Cooling curve for a liquid.

there is a change in the curve; it suddenly levels off, showing that the temperature does not change as the liquid is being cooled at the temperature marked t_f. Then, after a period of time, the temperature suddenly starts dropping again, although not quite at the same rate as before (the slope of the curve is not the same).

This sort of trend is especially easy to see when the data are plotted on a graph. The fact that a system shows different behavior in different regions is very interesting. What is happening to cause such a change? Sometimes, observation of the system gives a clue to the reason for the change. Other times, the pattern shown when experimental measurements are plotted on a graph is the first clue that there is something to be explained. Here, observation of the liquid while measurements are made would show that it is freezing (turning from a liquid to a solid) during the time the temperature remains unchanged. When all the liquid is frozen, further cooling lowers the temperature of the solid. The temperature t_f is the freezing point of the substance. What would you expect the curve to look like if you heated the solid at a steady rate and if the temperature levels off during the time the solid is melting?

This, incidentally, shows a limitation on the use of graphs to extrapolate, to predict what will happen outside the region measured. That is, the behavior of the system may be different in different regions. Therefore, graphs and equations representing physical situations may apply only within certain limits.

Graphs of physical measurements (measurements made on actual objects and systems) can have a variety of shapes. If you drop an object from a height and measure its position at intervals, you will get a set of data like that in Table 6.1. Plotting these points as distance traveled (ordinate) versus time (abscissa) and connecting the points in a smooth curve gives Figure 6.11. Look carefully at the curve. Notice that the curves goes through the origin; that is, at time 0 (start) the object has not yet fallen. At first it falls fairly slowly and the curve shows this by rising slowly. As time goes on (farther to the right on the curve), the line rises faster and faster, as the object falls, increasing distances in equal periods of time. We can say of the graph that the *slope* increases with time. We can say of the object that it is moving faster and faster, that is, acceler-

TABLE 6.1

Time (sec)	*Distance from Top* (m)
0	0
1	1
2	4
3	9
4	16

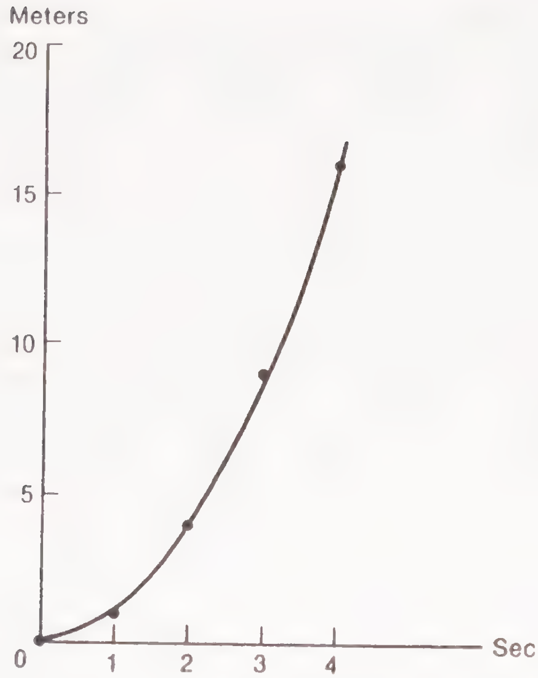


FIGURE 6.11 Graph of the distance an object falls versus time.

ating as time goes on. The rate at which the motion of the object (or the slope of the curve) changes is the subject of differential calculus.

Figures 6.12(a) and (b) shows other curved graphs. In Figure 6.13(a), as the quantity shown on the abscissa increases (going to the right), the quantity shown on the ordinate increases rapidly at first (curve goes up steeply). Then the increase slows down, and the change is rather slow (the curve rises slowly) at larger values of x.

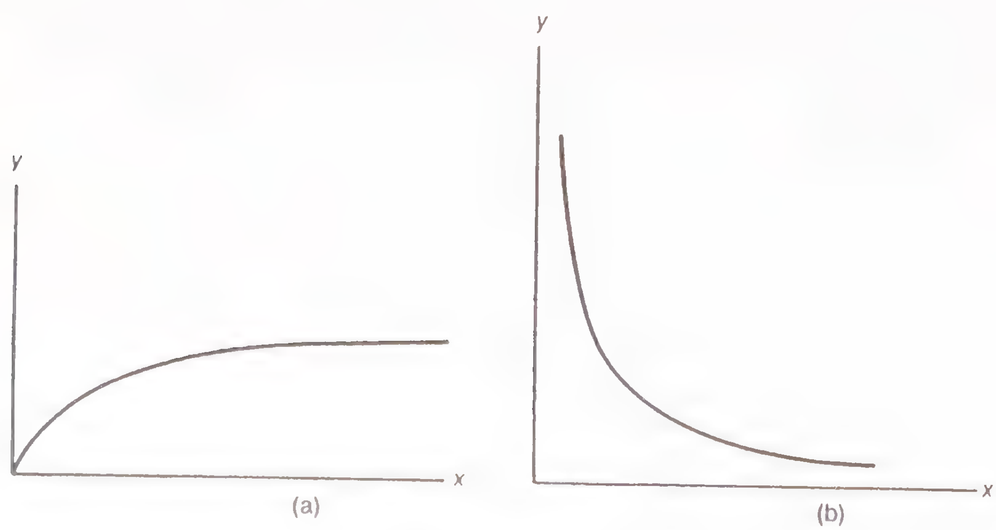


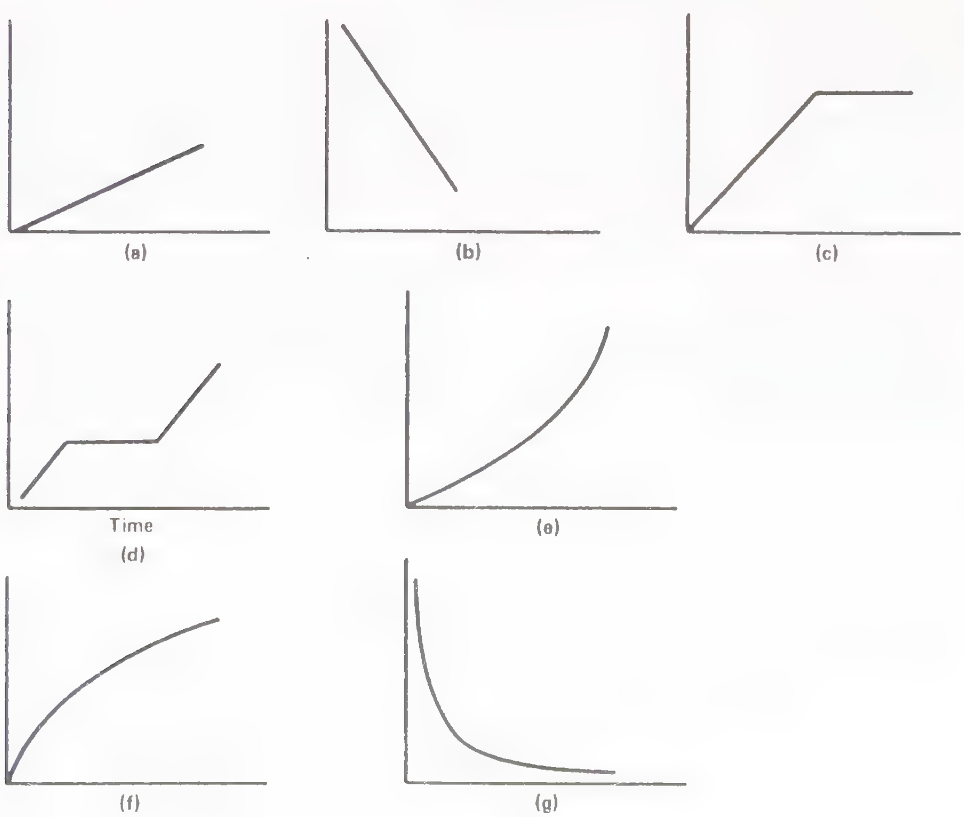
FIGURE 6.12

The curve in Figure 6.12(b) shows the common situation in which an increase in one quantity is accompanied by a decrease in another: the larger x (the farther to the right), the smaller y (the lower the curve). Here neither x nor y is ever 0. This curve represents an equation of the type $xy = k$. An example of this type would be the measurement of the pressure and volume of a gas: the larger the pressure, the smaller the volume, but a given sample of gas can never have a pressure or volume of 0.

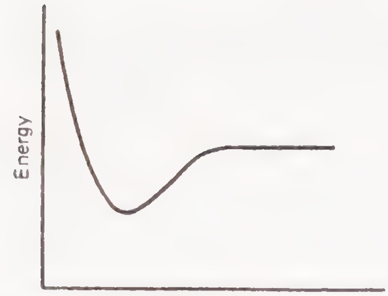
Note that these curves look almost like straight lines in some regions. In graphing experimental data, it is not safe to measure a few points close together, find that they seem to lie on a straight line, and assume that the graph will really be a straight line; you might have accidentally measured a region where there is little curvature but would have found a very different graph if your measurements had covered a wider range.

PROBLEM

6.3 Describe in words the behavior shown by each graph. If no other coordinates are given, consider the abscissa as x and the ordinate as y.




(h)



(i)

6.4. INTERPOLATION AND EXTRAPOLATION

A very important use of graphs is to let you read the value of one quantity that is associated with a particular value of another quantity, without having to measure every possible value. If you need to estimate a value beyond those measured, you can *extrapolate*, that is, continue the line on the graph (add *extra* length) and read off the needed value. For instance, if you have made measurements for 20 minutes and you want to estimate what the value would be after 1 hour, you would draw a continuation of the line out to 60 minutes and read off the value of the other quantity. (This assumes that the trend continues exactly as it started, something that is not always true.) If you need to find a value in between those you have measured, you would *interpolate*, that is, read the intermediate value from the graph.

Assume that you are riding along a highway that has very few markings to show where you are. You are traveling at a steady 55 miles/hr, and after a while you start wondering how far you have gone. It is simple enough to figure that after 1 hr you have traveled 55 miles, or after 30 min 22.5 miles, but what about the times in between? How far did you go in an hour and 25 min? On this long, dull stretch, it may be a diversion to do the arithmetic, but it is far easier to use a graph of distance versus time.

Knowing that at time 0 you had traveled 0 miles, at 60 min 55 miles, and at 120 min 110 miles, you can put these points on the graph and connect them with a straight line, Figure 6.13. Then you can look at your watch, see that you have been riding for 1 hr 25 min, find that point on your graph, and read off that it corresponds to 78 miles traveled. (This is only correct, of course, if you are traveling at a steady speed, but we already stated that as a condition. If your speed varies unpredictably, naturally it will not be possible to predict exactly where you will be at a given time. Nevertheless, if you *average* 55 miles/hr, the graph will tell you approximately how far you have gone.)

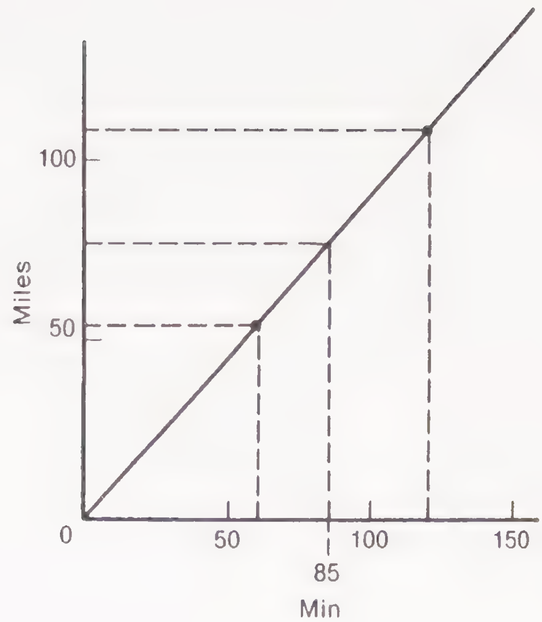


FIGURE 6.13 Graph of distance traveled versus time.

6.5. SLOPE

The slope of a curve is a measure of how fast the curve is rising or falling and, therefore, of how fast the quantity plotted is increasing or decreasing. The numerical value of the slope of a straight line is especially useful, because it is the proportionality constant m in the general equation for a straight line,

$$y = mx + b$$

where y is whatever quantity is plotted on the vertical axis, x is whatever quantity is plotted on the horizontal axis, and b is the *intercept*, the value of y when $x = 0$ (the place at which the line intercepts the vertical axis). For many systems, this constant m gives the value of a quantity of physical importance.

For a graph that is a straight line, the slope is determined by choosing two points on the line and measuring the change in the quantity plotted on the vertical axis and the change in the quantity plotted on the horizontal axis. A change is always measured as the final value minus the initial value.

$$\text{slope} = \frac{y_{\text{final}} - y_{\text{initial}}}{x_{\text{final}} - x_{\text{initial}}}$$

This means that the slope of a line that goes down as it goes to the right is negative (final value smaller than initial value). The slope of a line that goes up as it goes to the right is positive.

EXAMPLE 1

Calculate the slope of the line for each graph (Figure 6.14). Write the equation for each line.

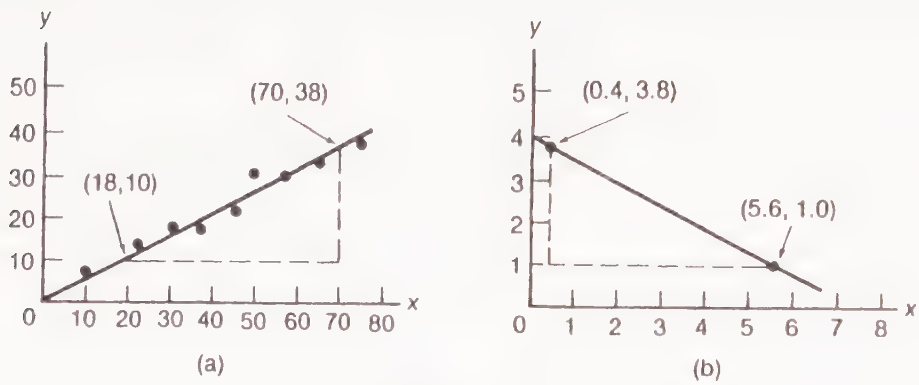


FIGURE 6.14

Mark two points on the line that are reasonably far apart. Do not simply choose two experimental measurements. From the graph, read the coordinates of each point.

(a) The coordinates of the two points are (70, 38) and (18, 10).

$$y_f = 38 \qquad y_i = 10 \qquad x_f = 70 \qquad x_i = 18$$

$$\text{slope} = \frac{(38 - 10)}{(70 - 18)} = \frac{28}{52} = 0.54$$

Since the line crosses the y axis at $x = 0$, the equation for the line is

$$y = 0.54x + 0 \qquad \text{or} \qquad y = 0.54x$$

(b) The coordinates of the two points are (0.4, 3.8) and (5.6, 1.0)

$$\text{slope} = \frac{(1.0 - 3.8)}{(5.6 - 0.4)} = -\frac{2.8}{5.2} = -0.54$$

The intercept is $y = 4$. The equation for the line is

$$y = -0.54x + 4$$

In scientific work one often plots the logarithm, the reciprocal, the square, or some other function of a measured quantity, rather than the quantity itself, in order to produce a graph that is a straight line. When drawing such a graph it is important to be especially careful in labeling the axes, to make clear exactly what is being plotted. A straight-line graph has a number of advantages. There is no uncertainty about the exact shape of the curve, as there might have been in Figure 6.5 in extending the curve to point *b*. Therefore, intermediate points can be read off the graph reliably. A straight line is useful when a numerical value for the slope is needed; it is much easier to measure the slope of a straight line than that of a curved line.

Sometimes the shape of a graph can be used to give a clue to the physical behavior occurring in an experiment. For instance, in a kinetics experiment the concentration of reactant is measured at various times t. In one possible type of reaction, the logarithm of the concentration is directly proportional to the time. In another possible type of reaction, the reciprocal of the concentration is proportional to the time. If you plot the logarithm of the concentration against time on one graph, and the reciprocal of the concentration against time on another, you can see which gives a straight line and therefore know which type of reaction is occurring.

If it is necessary to measure the slope of a graph that is not a straight line, a tangent must be drawn at the point of interest. A tangent is a straight line that just touches the curve at one point. The slope of that tangent line is the slope of the curve at that point.

PROBLEM

6.4 For all graphs in Problem 6.1 that are straight lines,

(a) Determine the slope of the line.

(b) Measure the value of the intercept (the point where the line crosses the vertical axis). Write the equation for the line, by substituting the values of the slope, m, and the intercept, b, into the equation $y = mx + b$.

---

two sides are equivalent, and remain so, as long as they are not changed in different ways.

Usually, in an equation containing letters, one wants to "solve for x," that is, find what value of the unknown or variable satisfies the equation. (Of course, this unknown or variable may be designated by any convenient letter or symbol, not just by x). A value of x satisfies the equation (is a *solution* or *root* of the equation) if, when it is substituted for x, the equation remains true. For instance, $x = 2$ is a correct solution of the equation $x + 3 = 5$, since 2 can be substituted for x and the equation will remain true.

$$2 + 3 = 5$$

$$5 \equiv 5$$

Any other number, such as 1, is not a correct solution. If 1 is substituted for x, the equation does not remain true, which proves that 1 is not a root of the equation:

$$1 + 3 \neq 5$$

$$4 \neq 5$$

In an equation that contains only one letter, the *unknown*, the letter can have only certain specific values, which are called *solutions* or *roots* of the equation. There will be as many roots as the highest power of the unknown. That is, if the unknown appears to the first power only, there is one root; if it is present to the second power, there are two roots; and so forth.

In an equation that contains two or more letters, the letters are known as *variables*, since their values are not fixed. If specific values are assigned to all but one of the letters, the equation is converted to one with a single unknown and can be solved for the value of that unknown. Alternatively, the equation can be solved for any one of the variables *in terms of* the others. That is, the solution will not be a single numerical value but an expression containing the other variables.

If

$$x + y = z$$

then

$$x = z - y$$

and

$$y = z - x$$

The following section discusses procedures for solving linear equations, equations in which no term appears to higher than the first power. (These are called "linear" because graphs of such equations are straight lines.) These procedures can be used for any equation in which the variable being solved for appears only to the first power, even if other variables appear to higher powers. Methods of solving equations containing the unknown raised to higher powers and methods of solving simultaneous equations will be presented in Chapter 9.

7.2. SOLVING LINEAR EQUATIONS

The first step in solving an equation for one variable (or unknown) is to isolate that variable. That means that all terms containing that variable are on one side of the equals sign and all terms that do not contain the variable are on the other side of the equals sign. (Remember: A term is a quantity added or subtracted. A factor is used to multiply something.)

A quantity can be removed from the side of the equation where it is not wanted by the inverse operation of the one that placed it there. That is, if the number is shown as added, it can be removed by subtraction (strictly speaking, by adding the additive inverse, which is the same number with the sign changed). If a number multiplies x, it may be removed by division. If a number divides x, appearing as the denominator of a fraction, it may be removed by multiplication. **For every operation performed, the same thing must be done to both sides.**

■ EXAMPLE 1

Solve for x.

$$x + 5 = 7$$

To isolate the x, the term $+5$ must be removed from the left side of the equation. This can be done by subtracting 5 (or adding -5, which amounts to the same thing). The 5 must be subtracted from *both* sides of the equation.

Subtracting 5 from both sides, then combining terms, gives

$$x + 5 - 5 = 7 - 5$$

$$x = 2$$

■

■ EXAMPLE 2

Solve for x.

$$27 = x - y$$

To isolate the x, the $-y$ must be removed from the right side of the equation. (It does not matter whether the x is on the left or the right; the procedure for separating terms containing x from terms that do not contain x is the same.) To remove the $-y$, it is necessary to add $+y$ *to both sides* of the equation.

$$27 + y = x - y + y$$

Grouping, we obtain

$$27 + y = x$$

■

If you look carefully at Examples 1 and 2, you may notice that it looks as if a quantity had simply been moved from one side to the other side with the sign changed:

$$x + 5 = 7$$
$$x = 7 - 5$$

and

$$27 = x - y$$
$$27 + y = x$$

This is indeed the effect of the operation performed. People who are experienced in solving equations usually think of the operation as "move it to the other side and change the sign" and do not write out the intermediate steps. Although you should write out all the steps until you feel very confident about what you are doing, you should be able to recognize what is being done if you see someone solving an equation without bothering to write all the steps.

For most equations, solving for x requires more than one step. Start by grouping terms that contain the unknown on one side, and terms that do not contain the unknown of the other side. Then perform such other operations as are necessary to obtain a value for the unknown.

■ EXAMPLE 3

$2x - 6 = 10.$

Group all terms that contain x on one side and all that do not contain x on the other. This requires adding 6 to both sides.

$$2x - 6 + 6 = 10 + 6$$
$$2x = 16$$

Now, since x is multiplied by 2, remove the 2 by the inverse operation, division. That is, divide both sides by 2.

$$\frac{2x}{2} = \frac{16}{2}$$

$$x = 8$$

Check by substituting 8 for x in the original equation:

$$2(8) - 6 = 10$$

$$16 - 6 = 10$$

$10 = 10$ Therefore, the value $x = 8$ is a correct solution for the equation. ■

■ EXAMPLE 4

$5x - 7 = 2x + 2$.

To group all terms containing x on one side, subtract $2x$ from both sides. To group all terms that do not contain x on the other side, add 7 to both sides.

$$5x - 7 - 2x + 7 = 2x + 2 - 2x + 7$$

$$3x = 9$$

Then divide both sides by 3.

$$\frac{3x}{3} = \frac{9}{3}$$

$$x = 3$$

■

■ EXAMPLE 5

$2x + 4y = 8$. (a) Solve for x; (b) solve for y.

(a) To solve for x, group all terms that do not contain x on the same side; this includes terms containing y.

$$2x + 4y - 4y = 8 - 4y$$

$$2x = 8 - 4y$$

Then divide by 2, being careful to divide every term.

$$\frac{2x}{2} = \frac{8}{2} - \frac{4y}{2}$$

$$x = 4 - 2y$$

(b) The same sequence of steps is used to solve for y. All terms that contain y are grouped on one side, and all terms that do not contain y are grouped on the other side.

$$2x + 4y = 8$$

$$4y = 8 - 2x$$

$$\frac{4y}{4} = \frac{8 - 2x}{4}$$

$$y = \frac{4 - x}{2} \quad \text{or} \quad 2 - \frac{x}{2} \quad \text{or} \quad 2 - 0.5x$$

The result can be expressed in any of the ways shown. ■

From here on, the individual addition, subtraction, multiplication, and division steps will not be shown. If you need to see them to verify the results shown, be sure to write them out.

PROBLEMS

For all problems, check your answer by substituting into the original equation.

7.1 Solve for x.

(a) $x + 3 = 7$
*(b) $x - 3 = 7$
(c) $x + y = 27$
(d) $2x = 32$
(e) $2x = 6y$
*(f) $2x - 7 = x + 6$
(g) $5x + 9 = 2x + 30$
(h) $2.5x + 3 = 15 - 1.5x$
(i) $4x + 0.2 = 6.6$

7.2 Solve for x.

(a) $3x + y = 27$
*(b) $5x - 2y = 15$
(c) $3x + 6y + 4 = 10$
(d) $x + yz = 0.5$

*(e) $ax + yz = b$ (f) $ax - 3 = yz$
(g) $ax - y = bz$ (h) $axy - bz = 0$
(i) $2xz - 3ay = 0$

7.3 Solve all parts of Problem 7.2 for y.

7.4 Substitute the values given into the equation and solve for the remaining unknown.
(a) $[H^+][OH^-] = 10^{-14}$; $[H^+] = 10^{-7}$
(b) $[H^+][OH^-] = 10^{-14}$; $[OH^-] = 10^{-13}$
(c) $[Mg^{2+}][OH^-]^2 = 1.2 \times 10^{-11}$; $[OH^-] = 6.0 \times 10^{-3}$

7.3. EQUATIONS CONTAINING FRACTIONS

It is very common to encounter equations that contain fractions. Some examples are

$$5 + \frac{2x}{3} = 9$$

$$\frac{8}{x} = 2$$

$$x + \frac{1}{2} = 2$$

If fractions occur only in terms that do not contain variables, as in the last example, the fractions may either be left as fractions or converted to decimals, and may be used like any other numbers. More often, however, variables are present in the fractions, and it is necessary to *clear the fractions*, that is, to convert the equation to one that does not contain the unknown as part of any fraction.

Since a fraction indicates that the numerator is to be divided by the denominator, the denominator(s) can be removed by the inverse operation, multiplication. To avoid changing the meaning of the equation, the same multiplication must be done on every term. Therefore, **the first step in solving an equation involving fractions is to clear the fractions by multiplying every term in the equation by the denominators of all fractions.**

Two variations on this rule can sometimes be used to save arithmetic.

Multiply all terms by the lowest common multiple of the denominators of the fractions.

OR Multiply by the denominators of all fractions that have the variable as part of the fraction. Terms that do not contain the variable remain as fractions.

EXAMPLE 6

Solve for x.

$$5y + \frac{2x}{3} = 9$$

Multiply *all terms* by 3.

$$3\,(5y) + \cancel{3}\left(\frac{2x}{\cancel{3}}\right) = 3\,(9)$$

$$15y + 2x = 27$$

Then

$$2x = 27 - 15y$$

$$x = \frac{27 - 15y}{2}$$

Check the answer by substituting into the original equation.

$$5y + \frac{2\left(\frac{27 - 15y}{2}\right)}{3} = 9$$

$$5y + \frac{27 - 15y}{3} = 9$$

$$5y + 9 - 5y = 9$$

EXAMPLE 7

Solve for v.

$$\frac{8}{v} = 2$$

Multiply both sides by v.

$$\cancel{v}\left(\frac{8}{\cancel{v}}\right) = v\,(2)$$

$$8 = 2v$$

$$4 = v$$

EXAMPLE 8

Solve for a.

$$\frac{a}{3} + \frac{b}{5} = 6$$

Multiply all terms by both denominators.

$$\cancel{3}\,(5)\left(\frac{a}{\cancel{3}}\right) + 3\,(\cancel{5})\left(\frac{b}{\cancel{5}}\right) = 3\,(5)(6)$$

$$5a + 3b = 90$$

$$5a = 90 - 3b$$

$$a = \frac{90 - 3b}{5} = \frac{90}{5} - \frac{3b}{5} = 18 - 0.6b$$

Any of the ways of showing the right side is correct, but the last is probably the most useful. If the value of b is found by some other measurement, it can easily be substituted and the numerical value of a calculated.

EXAMPLE 9

Solve for T.

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{T}$$

Multiply all terms by all three denominators.

$$\cancel{a}bT\left(\frac{1}{\cancel{a}}\right) + a\cancel{b}T\left(\frac{1}{\cancel{b}}\right) = ab\cancel{T}\left(\frac{1}{\cancel{T}}\right)$$

$$bT + aT = ab$$

$$T\,(a + b) = ab$$

$$T = \frac{ab}{a + b}$$

EXAMPLE 10

Solve for a.

$$\frac{a}{9.0} + \frac{n}{24} = \frac{224}{11{,}200}$$

This can be cleared of fractions by multiplying by all three denominators. However, the right side, which does not contain a variable, can be converted into a decimal.

$$\frac{a}{9.0} + \frac{n}{24} = 0.0200$$

$$9.0\,(24)\left(\frac{a}{9.0}\right) + 9.0\,(24)\left(\frac{n}{24}\right) = 0.0200\,(9.0)(24)$$

$$24a + 9.0n = 4.3$$

$$24a = 4.3 - 9.0n$$

$$a = \frac{4.3 - 9.0n}{24}$$

$$= 0.18 - 0.38n$$

In Example 10, it would be possible to save arithmetic by waiting to multiply the right side until after all possible canceling had been done:

$$24a + 9.0n = 0.0200\,(9.0)(24)$$

$$24a = 0.0200\,(9.0)(24) - 9.0n$$

$$a = \frac{0.0200\,(9.0)(\cancel{24})}{\cancel{24}} - \frac{9.0n}{24}$$

$$= 0.18 - 0.38n$$

PROBLEMS

7.5 Solve for x.

*(a) $\dfrac{x + 3}{8} = 2$ (b) $\dfrac{x}{2} = 8$

(c) $\dfrac{8}{x} = 2$ (d) $\dfrac{x}{2} = 15 + y$

*(e) $\dfrac{a}{x} = b$ (f) $\dfrac{y}{x} = a$

(g) $\frac{1}{x} = 7ab^2$

(h) $\frac{ax}{b} = yz$

(i) $\frac{2x}{3} = 72$

*(j) $\frac{x}{-40 - 32} = \frac{5}{9}$

(k) $\frac{x}{77 - 32} = \frac{5}{9}$

(l) $\frac{25}{x - 32} = \frac{5}{9}$

(m) $\frac{-40}{x - 32} = \frac{5}{9}$

7.6 Solve for x.

(a) $\frac{x}{2} + \frac{y}{3} = 15$

*(b) $\frac{8}{x} + 2 = 0$

(c) $\frac{4}{x} + 2 = 10$

(d) $\frac{1}{x} + \frac{1}{y} = 0$

*(e) $\frac{1}{x} + \frac{1}{y} = \frac{1}{T}$

(f) $\frac{x}{z} + \frac{a}{b} = c$

(g) $\frac{x}{16} + \frac{n}{27} = 0.020$

(h) $\frac{x}{12} + \frac{f}{55} = \frac{3.36}{22.4}$

(i) $\frac{1}{x} + 3 = y$

(j) $\frac{1}{x} + \frac{1}{y} = 1$

7.7 Solve for x, including units as well as numbers in your answer. Treat the units as if they were letters.

*(a) $4\,\frac{\text{mol}}{\text{L}} = \frac{2\text{ mol}}{x}$

(b) $\frac{0.50\text{ g}}{x} = 78\,\frac{\text{g}}{\text{mol}}$

(c) $0.50\,\frac{\text{mol}}{\text{L}} = \frac{x}{2.0\text{ L}}$

*(d) $0.89\,\frac{\text{g}}{\text{cm}^3} = \frac{x}{10\text{ cm}^3}$

(e) $0.80\,\frac{\text{g}}{\text{cm}^3} = \frac{20\text{ g}}{x}$

(f) $\frac{x}{3.0\text{ L}} = \frac{1\text{ atm}}{2.0\text{ L}}$

(g) $\frac{2.0\text{ atm}}{x} = \frac{1\text{ atm}}{5.0\text{ L}}$

7.8 Substitute the values given into each equation and solve for the remaining unknown.

*(a) $\frac{ab^2}{c} = 2.0;\qquad b = 3.0,\quad c = 15$

(b) $\frac{ab^2}{c} = 2.0;\qquad a = 4.0,\quad b = 7.0$

(c) $\dfrac{a}{b} + \dfrac{c}{d} = 5; \qquad a = 2, \quad b = 4, \quad c = 2$

*(d) $\dfrac{[H^+][F^-]}{[HF]} = 3.5 \times 10^{-4}; \qquad [F^-] = 10^{-2}, \quad [HF] = 10^{-2}$

Note: See the results of part (d). What is $[H^+]$ whenever $[F^-] = [HF]$?

(e) Same equation as part (d); $[H^+] = [F^-] = 1.9 \times 10^{-2}$

(f) Same equation as part (d);

$$[H^+] = 1.8 \times 10^{-1}, \qquad [F^-] = 5.0 \times 10^{-5}$$

(g)
$$\frac{[H^+][CN^-]}{[HCN]} = 5 \times 10^{-10};$$
$$[CN^-] = 10^{-5}, \quad [HCN] = 10^{-1}$$

7.4. WRITING EQUATIONS

Scientific work involves description not only of *what* occurs but (especially) of *how much.* Measurements and quantitative descriptions are essential to a meaningful scientific discussion. These quantitative descriptions can be expressed in words, but mathematical equations are preferable for a number of reasons.

1. The mathematical equation is shorter than the description in words. Compare the sentences and equations in the following sections.
2. It is easier to express a precise meaning mathematically.
3. If calculations or predictions are to be made, an equation is needed.
4. The language of mathematics is the same throughout the world. Sentences are subject to problems in translation from one language to another.

To translate a sentence in words into one in mathematical notation, the following procedures are used:

1. Assign a mathematical symbol to each quantity defined. It is convenient to use a single letter, but a descriptive word or phrase may be used. Write down what each symbol stands for.
2. Use the notation of mathematics to show the relation between these symbols.
3. Use an equals sign to show what is equivalent to what. The equals sign is the verb of the sentence, especially when the verb is a form of "to be."

Let us see how this works in a trivial example. Write an equation for calculating the net weight of the contents of a package, knowing the total weight t and the weight of the packaging materials p. The net weight n will be (=) the difference (subtract) between t and c.

$$n = t - p$$

Similarly, write an equation for the mass of an alloy (mixture) of aluminum and nickel. The mass of the alloy is the sum of the masses of the two elements. Let m = mass of the alloy, a = mass of aluminum, and n = mass of nickel in the alloy. As before,

$$m = a + n$$

This looks obvious, but it might have looked less obvious if the question had been phrased: "Give an equation for the mass of aluminum in a 2.00-g sample of an aluminum–nickel alloy." Here the total mass of the alloy is known, so the equation becomes

$$2.00 \text{ g} = a + n$$

This equation can be solved for a, the mass of aluminum.

$$a = 2.00 \text{ g} - n$$

This makes sense; the mass of aluminum is the difference between the total mass and the mass of nickel.

You might find it helpful in a problem like this to draw a picture (Figure 7.1). Although the aluminum and nickel are actually mixed in the alloy, the picture will be easier to relate to the equation if they are drawn as separate parts of a bar of metal.

Sometimes it is necessary to sort out the information given and perhaps to write it in two or more verbal statements. Then each of these can be translated into a mathematical statement.

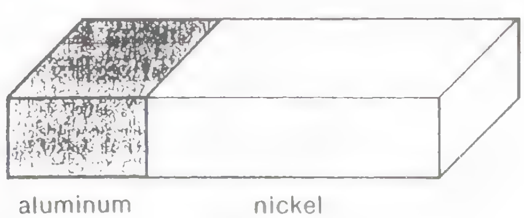


FIGURE 7.1

■ EXAMPLE 11

There exist two numbers such that their sum is 16 and their product is 63. What are the numbers?

Here, two ideas are conveyed. Each can be stated as a sentence:

The sum of two numbers is 63.

The product of the same two numbers is 63.

Again, it is convenient, although not necessary, to use letters as the symbols for the two numbers, and then to use these symbols in translating the sentences into equations. Let $x =$ one of the numbers; let $y =$ the other number. Then

$$x + y = 16 \quad \text{(The sum of the two numbers is 16.)}$$

and

$$xy = 63 \quad \text{(The product of the two numbers is 63.)}$$ ■

■ EXAMPLE 12

Write an equation for the sentence, "Tom is twice as old as Ann." This is not a statement about Tom and Ann in general but is a statement about their ages. The sentence could be rephrased, "The age of Tom is twice the age of Ann."

Let $t =$ Tom's age. Let $a =$ Ann's age. Then

$$t = 2a \quad \text{since "twice" means multiplication by 2}$$ ■

■ EXAMPLE 13

"In 5 years, Tom will be 1.5 times as old as Ann." Rephrase, "Tom's age in 5 years will be 1.5 times Ann's age in 5 years." Note that Ann as well as Tom will be 5 years older. The fact that the ages of both increase is implied in the situation described, even though it is not stated specifically. It is important to consider the physical reality of the situation for such clues.

If $t =$ Tom's age now, $t + 5 =$ Tom's age in 5 years. If $a =$ Ann's age now, $a + 5 =$ Ann's age in 5 years. Then

$$t + 5 = 1.5\,(a + 5)$$ ■

PROBLEMS

7.9 Write an equation to express each of the following.
- (a) The sum of two numbers is 23.
- (b) The difference between two numbers is 6.
- (c) The product of two numbers is 70.
- *(d) One number is three times as big as another.
- (e) One number is 5 more than another.

7.10 Write an equation for each.
- *(a) If Smith, Brown, and Jones pool their funds to form a partnership, what is the total capital (money available)?
- (b) Tom and Huck are painting a fence. How many feet of fence will get painted?
- (c) How much meat must you buy for a dinner party if you figure 0.5 lb for each guest?
- *(d) How many bottles of wine will you need if you figure a bottle for every three guests?
- (e) How many rose bushes should you buy if you plan to put three in each flower bed?
- (f) What mass of iron can be obtained from an ore if there is 1 kg of iron in every 20 kg of ore?
- (g) How many moles (a mole is a chemical unit of measure) of hydrogen will be formed in a process in which 1 mole of hydrogen is formed for every mole of zinc used?
- (h) How many moles of hydrogen are formed if 2 moles of hydrogen are formed for 1 mole of oxygen formed?
- (i) How many chloride ions are present in a solution if two chloride ions (Cl^-) are formed from each zinc chloride ($ZnCl_2$) unit that dissolves?
- (j) How many phosphoric acid (H_3PO_4) molecules are present in a solution if for every x molecules originally put in, y molecules have broken up to form H^+ and $H_2PO_4^-$ ions?

7.5. READING EQUATIONS

It is possible to draw inferences from algebraic equations even without putting in numbers to see the results. Perhaps your teacher has written an equation on the board and said, "From this you can see that. . . ." It is entirely possible, with a little practice, to see a great deal from an equation.

It is useful to define the concepts *dependent variable* and *independent variable*. Let us consider the equation $x + y = z$. This tells us that the

sum of two quantities, x and y, must be a quantity, z. For instance, if $x = 2$ and $y = 4$, z must equal 6. Here two of the variables, such as x and y are independent variables, since each may have any value and still leave the equation correct; z is then a dependent variable, since its value depends on those chosen for x and y. It is equally possible for x and z to be the independent variables, leaving y as the dependent variable or for y and z to be independent and x dependent. For example, if $x = 2$ and $z = 11$, y must be 9. We can choose whichever variables we want as independent, but **one of the variables must be dependent; it is not possible for all variables in an equation to be independent**.

Consider the simple equation $y = 2x$. Either variable, x or y, but not both, can be independent. The other is then dependent. If we choose x as the independent variable, the value of y depends on the value of x. We say that y varies with x, to mean that a change in x results in a corresponding change in y. How will the value of y vary with x for this equation? If x increases, y must increase, not at the same rate but twice as fast, as a result of the coefficient 2. If you cannot see that this is so, try assigning some values to x and calculate y.

x (assigned arbitrarily)	1	2	3	4	5
y (calculated)	2	4	6	8	10

7.6. EQUATIONS SHOWING PROPORTIONALITY

In the relationship described by the equation $y = 2x$, y is said to be *directly proportional* to x. That is, y changes in proportion to x and in the same direction, going up when x goes up and down when x goes down. Direct proportionality is sometimes shown by the expression

$$y \propto x \qquad \text{read "}y\text{ is proportional to }x\text{"}$$

Such an expression is converted into an equation by replacing the proportionality sign, $\propto$, by an equals sign, $=$, and multiplying by a constant (generally called the *proportionality constant*). The numerical value of this constant can be determined experimentally.

$$y \propto x \qquad \text{becomes} \qquad y = kx$$

Equations for proportionality can appear in other forms. The equation

$$y = 2x$$

can be rearranged to give

$$\frac{y}{x} = 2$$

How would you interpret this equation? The fraction y/x must always have the same value, 2. Therefore, any change in x must be accompanied by a change in y *in the same direction*. That is, if x increases, y must also increase. Try the values of x and y from the table in Section 7.5; they will still make the equation true. This, then, is another way to show that two quantities are directly proportional.

Another type of proportionality is represented by the equation

$$y \propto \frac{1}{x} \qquad \text{or} \qquad y = \frac{k}{x}$$

where y is inversely proportional to x. If x increases, y must decrease, and conversely a decrease in x means an increase in y. The larger the denominator of a fraction, the smaller the entire fraction. (If you have trouble seeing that this is so, think of cutting an object such as a pie into pieces. If the object is cut into more pieces, each piece will be smaller. The fraction 1/4 is smaller than 1/2, since it represents the cutting into four pieces rather than two. Another approach would be to calculate the decimal value of a few fractions, such as 1/4, 1/2, and 1/10. You will see that, as the denominator of the fraction increases, the value of the fraction decreases.)

The equation for inverse proportionality can be rearranged to

$$xy = k$$

where k is a constant (always has the same value).

If an increase in x as the denominator of a fraction means a decrease in y, what can we say about the equation as rearranged? It should be clear that the same thing is true; an increase in x requires a decrease in y so that their product will not change. If, for instance, $k = 20$, let us set up a table of possible values of x and y (Table 7.1). Notice that in order for

TABLE 7.1

x	y	$xy = k$
1	20	20
2	10	20
4	5	20
5	4	20
10	2	20

their product to remain constant, the values of x and y do *not* change by the same amount. When x goes from 1 to 2, it changes by 1 unit while y must change by 10 units. In going from 4 to 5, x again changes by 1 unit, but y this time changes by only 1 unit, from 5 to 4.

Inverse proportionality is encountered very frequently in science. That is, there are many situations in which an increase in one measurement must be accompanied by a decrease in another. For example, in measurements of a gas at constant temperature, increasing the pressure pushes the molecules closer together so that the gas occupies less volume. The situation is described by *Boyle's law*

$$PV = \text{constant}$$

or

$$P = \frac{k}{V}$$

where P is the pressure and V is the volume. (This equation applies only to gases, not to liquids and solids. Only in gases is there room between the molecules so that they can move closer together.)

Another example describes solutions of relatively insoluble ionic compounds like silver chloride, AgCl:

$K = [Ag^+][Cl^-]$ Where K is a constant, $[Ag^+]$ is the concentration of silver ion in solution, and $[Cl^-]$ is the concentration of chloride ion in solution.

Here, an increase in the concentration of one ion, as by adding another source of Cl^-, causes a decrease in the concentration of the other; some Ag^+ leaves the solution to form more solid AgCl.

One variable may vary as a higher power of another.

$y = kx^2$ y is directly proportional to the square of x.

$y = \dfrac{k}{x^2}$ y is inversely proportional to the square of x.

In these the changes are in the same direction as when the first power of x is used but the size of the change is much greater. An example of such dependence on the square of a variable can occur in certain chemical reactions, where the rate is directly proportional to the square of the concentration of one of the substances reacting.

rate $= kc^2$ where c is the concentration of the reactant

There are many examples of effects that are inversely proportional to the square of the distance between two objects. For example, the amount of light reaching an object of a given size varies inversely as the square of the distance between the object and the light source.

$$\text{amount of light} = \frac{k}{d^2}$$

If the object is moved twice as far from the light source, it will receive $1/(2)^2$, or 1/4, of the amount of light.

$$\frac{k}{(2d)^2} = \frac{k}{4d^2} = \frac{1}{4}\frac{k}{d^2}$$

An equation for describing a rate change often shows one quantity proportional to the logarithm of the other. (See Chapter 8.)

$$x \propto \log y \qquad \text{or} \qquad x = k \log y$$

Logarithms tend to seem to most poeple more like mathematical trickery than like anything having to do with physical reality. However, it is frequently found that the rate of a change is directly proportional to the amount (strictly speaking in most situations, the concentration) of starting material present. Mathematically, the rate of change is given by a differential. Converting such a differential equation to an algebraic equation (integration) gives an equation with a logarithmic term in it.

In all the equations discussed so far, there has been only one independent variable and, necessarily, one dependent variable. In practice, in most situations there are a number of things that can be varied. Does this mean that the types of variation considered so far are rare exceptions? Not at all. Even where there are many possible variables, it is still useful to consider a situation where most are held constant, and we are, in effect, looking at the result of changing only one of the variables.

Equations for physical phenomena often involve several kinds of proportionality. *Coulomb's law* describes the force between two charged particles by the equation

$$F = K\,\frac{q_1 q_2}{r^2}$$

where q_1 and q_2 are the charges on particles 1 and 2, respectively; K is a constant whose numerical value depends on the surroundings and on the units used; and r is the distance between the particles. The equation for Coulomb's law shows that F is directly proportional to q_1 and q_2 and also inversely proportional to the square of the distance between them.

We can estimate the kinds of changes that occur when only one or two of the variables are changed. For example, consider two ions (charged particles), of equal size but having different charges, such as sodium ion, Na^+, and calcium ion, Ca^{2+}, each in the vicinity of ions of opposite charge, such as Cl^-. The charges indicated in the superscript are the charges used in the equation. How does the force of attraction between Na^+ and Cl^- compare with the force of attraction between Ca^{2+} and Cl^-? Since the sodium ion and the calcium ion are essentially the same size, the distance r between the centers is the same both times. If they are measured in the same medium, say, air, the constant K is the same. The negative charge, -1, is also the same. Therefore, the only variables are the size of the positive charge, q_1, and the force, F. The force F is directly proportional to q_1 so that if q_1 goes from $+1$ to $+2$, the force must double. In other words, the attraction between a Ca^{2+} ion and a Cl^- ion is twice as great as that between an Na^+ ion and a Cl^- ion.

It would be possible to use the same equation to make a comparison of the change in force of attraction if the distance between the ions changes, but the charges are the same. For instance, one compound contains Cl^- and Ca^{2+} ions, radius 0.099 nm. Another compound contains Cl^- and Mg^{2+}, radius 0.066 nm. The distance between atoms or ions is considered to be that between the centers of the atoms or ions. The total distance between centers is the sum of the radii of the two atoms or ions. (Cl^- has a radius of 0.181 nm.) Since the force of attraction is inversely proportional to the square of the distance between the charged particles, the smaller the ions, the greater the force of attraction. Therefore the force of attraction is larger for the compound containing Mg^{2+} than for that containing Ca^{2+}.

What would have happened in the calculation if there had been more than one independent variable? For example, instead of comparing Mg^{2+} with Ca^{2+}, we might wish to compare Mg^{2+} (radius 0.066 nm) with Na^+ (radius 0.099 nm). This time both the charge and the distance vary. It is still possible to make a qualitative estimation of the difference without doing the complete calculation. When the charge on the ion is increased from $+1$ for Na^+ to $+2$ for Mg^{2+}, the force of attraction is doubled. The distance between opposite charges is *smaller* for the smaller Mg^{2+}. Therefore, the force of attraction is even *larger*, since it is *inversely* proportional to the square of the distance between the charges. If the two effects are combined, the force of attraction between Mg^{2+} and Cl^- is then *considerably more than twice as great* as the force of attraction between Na^+ and Cl^-.

To summarize the calculations with Coulomb's law, an equation involving several variables can sometimes be considered in the same way as an equation with only one independent variable by selecting conditions where some of the possible variables are fixed (effectively becoming

constants for the time being). Then estimates can be made of the extent to which the dependent variable varies with changes in the one remaining independent variable.

PROBLEMS

7.11 Describe in words what happens in each case. k is constant.

*(a) If $PV = k$, what happens to P if V increases?

(b) If $V = kT$, what happens to V as T increases?

(c) If $PV = kT$, what happens to P if V does not change, but T decreases?

(d) If $E = IR$, what happens to I if R increases with no change in E?

(e) If

$$M = \frac{\text{mol}}{V}$$

what happens to M if V is increased with no change in "mol"?

(f) In the equation for part (e), what happens to "mol" if M is increased with no change in V?

7.12 Compare the equations

$$E = IR \qquad \text{and} \qquad P = I^2R$$

(a) What happens to E if I is increased with no change in R?

(b) What happens to P if I is increased with no change in R?

(c) If I is doubled, do both E and P change by the same amount? Explain.

7.13 *(a) If $x + y = 27$, what happens to x if y increases? Are x and y proportional?

(b) If $x - y = 2$, what happens to x if y decreases?

7.14 In the equation

$$a = \frac{V_t - V_0}{t}$$

V_0 is constant.

(a) What is the effect on a if both V_t and t decrease?

(b) If $V_0 > V_t$ and t is positive, what is the sign of a?

7.15 The fundamental frequency, v, of a vibrating string (as on a violin or piano) of radius r and length l held at tension T is given by

$$v = \frac{1}{2rl}\sqrt{\frac{T}{\pi d}}$$

How is the frequency v affected by
(a) using a thinner string?
(b) using a longer string?
*(c) tightening the string?

7.16 In the equations that follow,
To what term, if any, is the term on the left directly proportional?
To what is it inversely proportional?
Does the equation show no proportionality?

*(a) $D = \dfrac{C_{aq}}{C_{org}}$ (b) $I = \dfrac{E}{R}$

*(c) $H = E + PV$ (d) $W = Pt$

(e) $a = \dfrac{V_t - V_0}{t}$ (f) $T = 2\pi\sqrt{\dfrac{l}{g}}$

(g) $Q = \dfrac{1}{3}Nmv^2$ (h) $MR = \dfrac{n^2 - 1}{n^2 + 2} \cdot \dfrac{M}{d}$

SOLUTIONS TO STARRED PROBLEMS

7.1(b) $x - 3 = 7$ To remove the -3, add $+3$ to both sides.

$x - 3 + 3 = 7 + 3$ Combine terms.

$x = 10$

(f) $2x - 7 = x + 6$ Subtract x from both sides to remove it from the right side.

$2x - x - 7 = x - x + 6$

$x - 7 = 6$

$x - 7 + 7 = 6 + 7$ Add $+7$ to both sides.

$x = 13$

7.2(b) $5x - 2y = 15$ Isolate the term in x on the left.

$5x - 2y + 2y = 15 + 2y$ Remove the $2y$ term from the left side.

$$5x = 15 + 2y$$

$$\frac{5x}{5} = \frac{15 + 2y}{5}$$

$$x = \frac{15 + 2y}{5} \quad \text{or} \quad \frac{15}{5} + \frac{2y}{5} = 3 + \frac{2y}{5}$$

(e) The procedure is identical to that for Problem 7.2(b), but here all terms use letters.

$ax + yz = b$ Isolate the term containing x.

$$ax + yz - yz = b - yz$$

$ax = byz$ Divide by a to remove it from the left.

$$\frac{ax}{a} = \frac{b - yz}{a}$$

$$x = \frac{b - yz}{a}$$

7.3(b) Solve for y.

$$5x - 2y = 15$$

Here y appears in a negative term. There are two ways to solve for a positive y. One is to add $2y$ to both sides, effectively moving the y term to the right side of the equation.

$$5x - 2y + 2y = 15 + 2y$$

$5x = 15 + 2y$ (Now solve the usual way.)

$$5x - 15 = 15 - 15 + 2y$$

$$= 2y$$

$$\frac{5x - 15}{2} = \frac{2y}{2}$$

$$\frac{5x - 15}{2} = y$$

The other procedure is to consider y to be multiplied by -1. To remove the -1, then, just divide all terms by -1, which has the effect of changing the signs of all terms. Notice that dividing by -1 has the same effect as multiplying by -1, and it is more common to say that one multiplies.

$$\frac{5x}{-1} - \frac{2y}{-1} = \frac{15}{-1}$$

$$5x = 15 + 2y$$

$$\frac{5x}{5} = \frac{15 + 2y}{5}$$

$$x = \frac{15 + 2y}{5} \quad \text{or} \quad \frac{15}{5} + \frac{2y}{5} = 3 + \frac{2y}{5}$$

(e) The procedure is identical to that for Problem 7.2(b), but here all terms use letters.

$$ax + yz = b$$ Isolate the term containing x.

$$ax + yz - yz = b - yz$$

$$ax = byz$$ Divide by a to remove it from the left.

$$\frac{ax}{a} = \frac{b - yz}{a}$$

$$x = \frac{b - yz}{a}$$

7.5(a) First clear of fractions by multiplying both sides by the denominator of the fraction, 8. Then solve as usual.

$$8\left(\frac{x + 3}{8}\right) = 8(2)$$

$$x + 3 = 8(2)$$

$$x = 16 - 3 = 13$$

(e) Clear of fractions by multiplying both sides by the denominator, x. Then solve.

$$x\left(\frac{a}{x}\right) = (b)x$$

$$a = bx$$

$$\frac{a}{b} = x$$

(j) This is the equation for finding the Celsius temperature equivalent to $-40°F$. Add the terms in the denominator before doing anything else.

$$\frac{x}{-40-32}=\frac{5}{9}$$

$$\frac{x}{-72}=\frac{5}{9}$$

Now the entire equation could be cleared of fractions, but it is faster and easier just to multiply by -72 and leave the 5/9. There is no real objection to fractions in terms that do not contain x. In fact, if both sides were multiplied by 9, they would have to be divided again by 9. (Try it and see.)

$$(-72)\,\frac{x}{-72}=\frac{5}{9}\,(-72)$$

$$x=\frac{5}{9}\,(-72)$$

$$=-40$$

7.6(b) You can either start by clearing of fractions or by moving the 2 to the other side of the equation. In either case you must, either as the first or second step, multiply all terms by x. Remember that zero times anything is zero.

$$x\left(\frac{8}{x}\right)+x\,(2)=x\,(0)$$

$$8+2x=0$$

$$2x=-8$$

$$x=\frac{-8}{2}=-4$$

Or else,

$$\frac{8}{x}+2=0$$

$$\frac{8}{x}=-2$$

$$8=-2x \qquad \text{multiplying by } x$$

$$-\frac{8}{2}=x \qquad \text{dividing by } -2$$

(e) First, clear of fractions by multiplying all terms by the denominators, xyT.

$$\cancel{x}yT\left(\frac{1}{\cancel{x}}\right) + x\cancel{y}T\left(\frac{1}{\cancel{y}}\right) = xy\cancel{T}\left(\frac{1}{\cancel{T}}\right)$$

$$yT + xT = xy$$ then group terms containing x

$$xT - xy = -yT$$ factor out the x

$$x(T - y) = -yT$$ divide by the coefficient of x

$$x = -\frac{yT}{T - y}$$

7.7(a) Treat the units as if they were single letters; clear of fractions by multiplying both sides by x. You can multiply by liters now or later.

$$x\left(4\,\frac{\text{mol}}{\text{L}}\right) = \cancel{x}\left(\frac{2\text{ mol}}{\cancel{x}}\right)$$

$$4x\,\frac{\text{mol}}{\text{L}} = 2\text{ mol}$$ Multiply by L and divide by 4 mol.

$$x = 2\,\cancel{\text{mol}} \times \frac{\text{L}}{4\,\cancel{\text{mol}}}$$

$$= 0.5\text{ L}$$

(d) Multiply both sides by 10 cm^3. The unit cm^3 will then cancel out, so there will be nothing more to do.

$$10\,\cancel{\text{cm}^3}\left(0.89\,\frac{\text{g}}{\cancel{\text{cm}^3}}\right) = 10\,\cancel{\text{cm}^3}\left(\frac{x}{10\,\cancel{\text{cm}^3}}\right)$$

$$8.9\text{ g} = x$$

7.8(a) First, substitute the numbers given into the equation.

$$\frac{ab^2}{c} = 2.0, \quad b = 3.0, \quad c = 15$$

$$\frac{a\,(3.0)^2}{15} = 2.0$$ Multiply both sides by 15.

$$9.0a = 2.0\,(15) = 30$$

$$a = \frac{30}{9.0} = 3.3 \quad \text{or} \quad 3\,\frac{1}{3}$$

(d) Substitute in the values given.

$$\frac{[H^+][F^-]}{[HF]} = 3.5 \times 10^{-4}; \qquad [F^-] = 10^{-2}, \quad [HF] = 10^{-2}$$

$$\frac{[H^+](10^{-2})}{10^{-2}} = 3.5 \times 10^{-4}$$

Now you can either multiply both sides by the denominator, 10^{-2}, or you can save work by canceling terms that appear both in the numerator and denominator. The 10^{-2} in the denominator will cancel with the 10^{-2} in the numerator.

$$\frac{[H^+](\cancel{10^{-2}})}{\cancel{10^{-2}}} = 3.5 \times 10^{-4}$$

$$[H^+] = 3.5 \times 10^{-4}$$

Any time that a term in the numerator equals a term in the denominator, they can be canceled, simplifying the problem. Here, whenever $[F^-] = [HF]$, the fraction $[F^-]/[HF] = 1$ and the equation will be left with the same value of $[H^+]$, 3.5×10^{-4}, whatever the value of the other two terms.

7.9(d) First, choose a symbol for each term in the equation and write down what each symbol stands for. Here we might unimaginatively choose x and y. Let x = one number; let y = the other number. Now translate the sentence into symbols. One number (x) is (=) three times (multiply by 3) as big as the other (y).

$$x = 3y$$

7.10(a) Here the sentence does not translate so directly into an equation. It can be rephrased, "The total capital is the sum of the funds put in by Smith, the funds put in by Brown, and the funds put in by Jones." This can be translated into an equation easily. Let s = the funds put in by Smith; let b = the funds put in by Brown; let j = the funds put in by Jones; let t = the total. Then the sentence can be written in symbols:

$$t = s + b + j$$

(d) Again some rephrasing is needed, but be careful. If one bottle of wine is to be served for every three guests, the equation must not indicate three bottles for every guest! A suitable rephrasing is, "The number of bottles of wine is 1/3 as large as the number of guests." Let b = number of bottles of wine; let g = number of guests.

$$b = \frac{1}{3}g$$

Another wording is, "There are three guests to every bottle."

$$g = 3b$$

7.11(a) If V increases, P must decrease in order for their product to remain constant as required by the equation $PV = k$.

7.13(a) In the equation $x + y = 27$, x and y are not directly proportional. However, if their sum is to remain constant, an increase in y must be accompanied by an equal numerical decrease in x.

7.15(c) Tightening the string means increasing the tension of the string. Therefore, the question is asking for the effect on v of an increase in T. In the equation,

$$v = \frac{1}{2rl}\sqrt{\frac{T}{\pi d}}$$

v is directly proportional to the square root of T. Therefore, tightening the string will cause an increase in the frequency; the size of the increase will be proportional to the square root of the increase in tension, so doubling the tension would cause v to be multiplied by 1.4 (the square root of 2). The increase in frequency means a note of higher pitch, which is indeed the effect of tightening a violin string.

7.16(a) In the equation $D = C_{aq}/C_{org}$, D is directly proportional to the numerator, C_{aq}, and inversely proportional to the denominator, C_{org}.

(c) Here H is directly proportional to the entire right side of the equation $(E + PV)$; H is not directly proportional to any one of the terms E, P, or V.

CHAPTER

8

LOGARITHMS

8.1. INTRODUCTION TO LOGARITHMS

Logarithms, or logs, appear in a large and surprising variety of usages. You can find roots of numbers by using logarithms. Logarithms occur in the equations that describe many chemical and physical processes. Graphs are sometimes drawn with logarithmic coordinates. On these graphs a coordinate has equal divisions marked 100, 1000, 10,000, 100,000, and so forth. Since these numbers are 1 unit apart on the graph, a very wide range of numbers can be put into a relatively small space. What are logarithms?

A logarithm is an exponent. Any base a can be raised to any exponent n,

$$a^n$$

Logarithms are exponents that are applied to one of two specific bases. Common logarithms use the base 10, so they are the exponent n of the number 10^n. Natural or Napierian logarithms use the base e, an irrational number having the value 2.71828. . . .

For convenience, the abbreviation "ln" is often used for natural logarithms and "log" for common logarithms. Natural logarithms can be related to common logs by the equation

$$\ln x = 2.303 \log x$$

Some calculators have log and antilog (INVerse log) functions. Even if you use a calculator to find logarithms, it is important to understand how to work with logarithms.

To write the logarithm of a number, you write only the exponent and not the base.

$$100 = 10^2$$

$$\log 100 = \log 10^2 = 2 \quad \text{"the Logarithm of 100 is 2."}$$

$$\log 1{,}000{,}000 = \log 10^6 = 6$$

The logarithm (base 10) of a number is the exponent to which 10 must be raised to achieve the number.

Since logarithms are exponents, the rules for calculations with logarithms are the same as the rules for calculations with exponentials.

1. To multiply, add the logarithms (or exponents).

$$\log xy = \log x + \log y$$

2. To divide, subtract the logarithms (or exponents).

$$\log \frac{x}{y} = \log x - \log y$$

3. To raise a number to a power, multiply the logarithm (or exponent) by the power. Since a root can be written as a fractional power, this also provides a way of finding roots.

$$\log x^n = n \log x$$

$$\log \sqrt[n]{x} = \log x^{1/n} = \frac{1}{n} \log x$$

TABLE 8.1

Number	*Exponential form*	*Log of the number*
1	10^0	0.000
2	$10^{0.301}$	0.301
3	$10^{0.477}$	0.477
4	$10^{0.602}$	0.602
5	$10^{0.699}$	0.699
6	$10^{0.778}$	0.778
7	$10^{0.845}$	0.845
8	$10^{0.903}$	0.903
9	$10^{0.954}$	0.954
10	$10^{1.000}$	1.000

Some trivial examples will illustrate the way in which the rules for using logarithms, combined with the numbers in Table 8.1, are used to perform calculations.

■ **EXAMPLE 1**

Multiply 2×4, using logs.

To multiply, add the logs. Therefore

$$\log (2 \times 4) = \log 2 + \log 4 = 0.301 + 0.602 = 0.903$$

or

$$10^{0.301} \times 10^{0.602} = 10^{0.903}$$

The product is 8, the number whose log is 0.903. ■

■ **EXAMPLE 2**

Find the square root of 9 using logs.

To find a root, divide the log:

$$\log 9^{1/2} = \frac{1}{2} (\log 9) = \frac{1}{2} (0.954) = 0.477$$

The number whose log is 0.477 is 3. ■

You may consider this unnecessary because you know that the square root of 9 is 3, but how would you find the fifth root? The only easy way is to use logarithms, whether you do it consciously or whether you use a calculator that is programmed to carry out the sequence of operations for you.

8.2. FINDING LOGARITHMS OF NUMBERS

To find the logarithm of a number that is an even power of 10, you can simply write the number in exponential form. The exponent is then the logarithm. To find the logarithm of a whole number between 1 and 10, you can use Table 8.1. How do you find logarithms of other kinds of numbers? The rules are

1. **Write the number in exponential notation with one digit before the decimal.**

2. **Find the logarithm of the coefficient, and find the logarithm of the exponential portion.**

3. **Since the logarithm of a product is the sum of the logarithms of the factors, the sum of the logarithms of the coefficient and of the exponential portion is the logarithm of the number.**

$$\log 500 = \log (5 \times 10^2)$$

$$= \log 5 + \log 10^2 \qquad \text{since multiplication requires adding the logarithms}$$

$$\log 10^2 = 2$$

$$\log 5 = 0.699$$

$$\log 500 = 0.699 + 2 = 2.699$$

This is reasonable, because

$$\log 100 = 2$$

$$\log 1000 = 3$$

500 is larger than 100 but smaller than 1000

log 500 is between 2 and 3

The part of the logarithm before the decimal, which is called the *characteristic*, merely shows the place of the decimal in the original number. The part of the logarithm after the decimal is called the *mantissa*, and it is this that locates the exact number. If the decimal in the original number had been in a different place, the mantissa would be the same with a different characteristic.

$$\log 5{,}000{,}000 = \log (5 \times 10^6)$$

$$= \log 5 + \log 10^6$$

$$= 0.699 + 6 = 6.699$$

Table 8.1 shows the logarithms of numbers with only one digit. If the coefficient of the number has more than one digit, you must use a table with more information, such as Table 8.2. Log tables show the mantissas of numbers between 1 and 9.99, but they omit decimal points, both from the numbers and from the logarithms. The numbers have a decimal

TABLE 8.2 FOUR-PLACE LOGARITHMS OF NUMBERS

n	0	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396

TABLE 8.2 (continued)

n	0	1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863
97	9860	9872	9877	9881	9886	9890	9894	9899	9903	9908
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996

point after the first digit. **The logarithms must always have a decimal point before the first digit shown.** The mantissa is shown to one more figure than the number of significant figures in the number.

A table of four-place logarithms like Table 8.2 shows two digits along the left side. The third digit is written across the top. Therefore, to find the logarithm of a number, find the first two figures in the number on the left side. Then read across the top of the table to the column corresponding to the third figure in your number. The place in the table where the horizontal and vertical columns cross contains the mantissa of the logarithm. The mantissa must be preceded by a decimal point.

EXAMPLE 3

Find the logarithm of (a) 39; (b) 3900.

First, write each number in exponential notation.

$$\log 39 = \log 3.9 \times 10^1$$
$$\log 3900 = \log 3.9 \times 10^3$$

Since both have the same coefficient, both will have the same mantissa. Look down the left side of the table for the 3, and then across to the column under 9. The mantissa for 3.9 is given as 59. This must follow the decimal, so it would correctly be written as 0.59.

$$\begin{aligned} \log 3.9 \times 10^1 &= \log 3.9 + \log 10^1 \\ &= 0.5911 + 1 = 1.5911 \\ \log 3.9 \times 10^3 &= 0.5911 + 3 = 3.5911 \end{aligned}$$

Properly, the mantissa is always positive. This creates a problem when finding the logarithm of a number such as 7.00×10^{-3}.

$$\begin{aligned} \log 7.00 \times 10^{-3} &= \log 7.00 + \log 10^{-3} \\ &= 0.845 - 3 \qquad \text{correct} \\ &= -2.155 \qquad \text{technically incorrect} \end{aligned}$$

This last way of writing logarithms is not really mathematically correct. The apparent mantissa shown, 0.155, is actually the mantissa of 1/7, not of 7. However, both ways of writing the logarithm give the same result in calculations, and it is far more convenient to combine the terms.

EXAMPLE 4

What is the logarithm of 0.0653?

Write the number in exponential form.

$$0.0653 = 6.53 \times 10^{-2}$$

$$\begin{aligned} \log 6.53 \times 10^{-2} &= \log 6.53 + \log 10^{-2} \\ &= 0.8149 - 2 \\ &= -1.1851 \end{aligned}$$

PROBLEMS

8.1 Find the log of each.

*(a) 10^{-2}

(b) 10^{9}

(c) 10^{130}

(d) 10^{-3}

8.2 Find the log of each.

*(a) 3.00×10^{9}

(b) 9.00×10^{3}

(c) 5.72×10^{3}

(d) 4.66×10^{200}

*(e) 466

(f) 5720

(g) 125

(h) 2.42

8.3 Find the log of each.

*(a) 3.00×10^{-9}

(b) 6.73×10^{-2}

(c) 2.81×10^{-5}

(d) 9.22×10^{-3}

(e) 5.49×10^{-13}

*(f) 0.00321

(g) 0.153

8.4 A student used a calculator to find the logarithms of some numbers. For each, tell whether the answer could be correct. Do not calculate the logarithm; just look for answers that could not possibly be correct.

*(a) Is $\log 3.42 = 2.5340$?

(b) Is $\log 3.42 = 2630$?

(c) Is $\log 0.342 = 0.4660$?

(d) Is $\log 0.342 = -0.4660$?

8.5 Calculate $\mathscr{E}$ for the following Nernst equation:

$$\mathscr{E} = \mathscr{E}^0 - \frac{0.0591}{n} \log Q$$

(a) $\mathscr{E}^0 = 1.10$, $n = 1$, $Q = 10^{-1}$.

(b) $\mathscr{E}^0 = 0.83$, $n = 2$, $Q = 10^{-2}$.

8.6 The quantity of the starting material still present was measured at various times, as a reaction occurred, giving the data shown.

Time	Mass
0	200 g
10 min.	100 g
20 min.	50.0 g
30 min.	25.0 g
40 min.	12.6 g

(a) Plot grams versus time.
(b) Calculate the logarithm of the number of grams (log g) for each measurement.
(c) Plot log g versus time.
(d) If either graph is a straight line, find the slope.
(e) Compare the graphs in parts (a) and (c)

8.7 The ultraviolet spectrum of a compound can be used to determine what is present or how much is present in a solution. Spectra are plotted in two ways: (1) the extinction coefficient, ε, a quantity proportional to the fraction of the light absorbed by the molecules, versus the wavelength of the light, or (2) the log of the extinction coefficient versus the wavelength.

Calculate log ε for each point. Draw graphs of ε versus wavelength and of log ε versus wavelength. [*Hint:* Round off numbers to two or three significant figures before plotting them.] Compare the graphs, commenting on the advantages and disadvantages of using the logarithm.

Wavelength (in nm)	ε
230	1.59
240	31.6
250	400
260	2000
270	5000
274	5620
280	3160

Wavelength (in nm)	ε
285	1600
290	640
300	16.0
304	3.16
310	7.94
315	2.00
320	1.26

8.3. ANTILOGS

To find a number whose logarithm is given (the antilog), reverse the process used to find the log. Look up the mantissa in the table, find what number this corresponds to, and use the characteristic to place the decimal.

EXAMPLE 5

Find the antilog of 2.4330. That is, find the number whose logarithm is 2.4330.

Rewrite the logarithm, separating the characteristic from the mantissa.

$$2.4330 = 2 + 0.4330$$

The antilog of 2 is 10^2. The antilog of 0.4330 must be found from the table. To use the table, remember:

The logarithms are the numbers inside the table.

The table does not show the decimal point before the mantissa.

The table does not show the decimal point after the first digit of the number.

Look for the number 4330 in the body of the table. Follow the line to the left to read the first two digits in the number, 27. Then look at the top of the column to find the third digit. Since 4330 appears in the column under 1, the third digit is 1 and the entire coefficient is 2.71.

$$\begin{aligned} \text{antilog } 2.4330 &= \text{antilog } (2 + 0.4330) \\ &= 2.71 \times 10^2 \end{aligned}$$ ■

Often you need the antilog of a mantissa that is between the numbers in the table. If you need to find the number whose log is 0.3550, you find that the table lists 3541 and 3560 but not 3550. Therefore, you must *interpolate*, that is, find the number between those shown. Find the fraction of the difference between the two numbers in the table

$$\frac{3550 - 3541}{3560 - 3541} = \frac{9}{19} \approx 0.5$$

Since 3550 is approximately 0.5 of the difference between 3541 and 3560, the antilog of 0.3550 would be 0.5 of the distance between 2.26 and 2.27, or 2.265. Some tables of logarithms have a supplementary "proportional parts" table at the side, to help you make such interpolations accurately.

If a logarithm is written as a negative number, it is necessary to use special care in finding the antilog. The mantissas given in the table are always positive; a negative mantissa corresponds to an entirely different number from that shown in the table. The antilog of 0.70 is 5, but the antilog of -0.70 is 0.2 (or 1/5). (Use the procedure of Example 6 to verify this.)

To convert a negative mantissa to a positive one that can be used with the log table, add 1.00 to the mantissa and subtract 1 from the characteristic. Since 1 has been both added and subtracted, the value of the logarithm will be unchanged, but the mantissa will now be positive.

The characteristic will be one digit more negative than the original value. It is a good idea to check the accuracy of this process by adding the resulting characteristic and mantissa to be sure that they do indeed add up to the original logarithm.

EXAMPLE 6

Find the number whose logarithm is -4.4750 (the antilog of -4.4750).

The first step is to convert the logarithm to one with a positive mantissa, by adding 1.00 to the mantissa and subtracting 1 from the characteristic.

$$\begin{array}{rrrl} -4.4750 = & -4 & + (-0.4750) & \\ & \underline{-1} & \underline{+1.0000} & \\ & -5 & +0.5250 & \textit{Check:} \quad -5 + 0.5250 = -4.4750 \end{array}$$

The antilog of -5 is 10^{-5}. The antilog of 0.5250 is found from the table to be 3.35. Therefore, the required number is 3.35×10^{-5}. ■

PROBLEMS

8.8 Find the antilog of each (the number whose log is given).

*(a) 10	(b) 2
(c) -1	(d) -8
*(e) 6.8035	(f) 0.3874
(g) 2.8209	(h) 5.6736
(i) 1.9456	

8.9 Find the antilog of each.

*(a) -6.2950	(b) -3.9626
(c) -0.8097	(d) -2.2510
(e) -12.6655	

8.4. pH

An especially common application of the use of logarithms in chemistry is the unit pH. This is a shorthand way of describing the concentration of hydrogen ion, $[H^+]$,* which can vary from about 10 to 10^{-15}. Logarithms work very well to compress this wide range of numbers into a more manageable range. Since most of the range involves negative exponents, the logarithms would be negative. To avoid having to write the minus signs, pH is defined as

$$\text{pH} = -\log [H^+]$$

To find pH, then, find the log of $[H^+]$ and change the sign. To find the molarity of hydrogen ion, $[H^+]$, given the pH, (1) change the sign, and (2) find the antilog.

$$[H^+] = \text{antilog}(-\text{pH})$$

EXAMPLE 7

Find the pH of a solution if $[H^+]$ is 3×10^{-4} M.
First, find the log of 3×10^{-4}. Then change the sign.

$$\begin{aligned} \log 3 \times 10^{-4} &= \log 3 + \log 10^{-4} \\ &= 0.48 + (-4) \\ &= -3.52 \\ \text{pH} = -\log [H^+] &= -(-3.52) \\ &= 3.52 \end{aligned}$$

Since the molarity is given only to one significant figure, two places are sufficient in the mantissa of the logarithm. The pH is rarely given to more than two or three places after the decimal.

* H^+ does not exist as a separate entity; it is always associated with something else. Many teachers prefer that you always write H_3O^+, indicating that the hydrogen ion is attached to a water molecule. Others feel that this notation introduces other errors. Although this book will use the simplest version, be careful to use the notation your teacher requires.

EXAMPLE 8

Find $[H^+]$ in a solution of pH 7.85.
First, change the sign of the pH.

$$pH = -\log [H^+]$$
$$-pH = \log [H^+]$$
$$-7.85 = \log [H^+]$$

Now find the number whose log is -7.85.

$$\begin{array}{rrr} -7.85 = & -7 & +(-0.85) \\ & -1 & +1.00 \\ \hline = & -8 & +0.15 \end{array}$$

The number whose log is -8 is 10^{-8}. The number whose log is 0.15 is found from the table to be 1.4. Therefore,

$$[H^+] = 1.4 \times 10^{-8} \text{ M}$$

PROBLEMS

8.10 Find the pH of each.
- *(a) $[H^+] = 10^{-3}$
- (b) $[H^+] = 1$
- (c) $[H^+] = 10^{-8}$
- (d) $[H^+] = 2 \times 10^{-5}$
- (e) $[H^+] = 7 \times 10^{-3}$
- *(f) $[H^+] = 0.3$
- (g) $[H^+] = 0.002$

8.11 Find $[H^+]$ for each solution.
- *(a) pH = 5
- (b) pH = 11
- (c) pH = 0
- *(d) pH = 3.2
- (e) pH = 9.44
- (f) pH = 6.4
- (g) pH = 0.49

8.12 Evaluate each answer. Without carrying out the calculation, decide whether each answer could be true.
- (a) If $[H^+] = 3.2 \times 10^5$ M, is pH 3.2?
- (b) If pH = 9.33, is $[H^+]$ 0.857 M?
- *(c) If $[H^+] = 4.2 \times 10^{-4}$, is pH 3.38 or 4.38?
- (d) Log 8.5 is 0.93. If $[H^+] = 8.5 \times 10^{-2}$, is pH 2.85?
- (e) Log 8.5 is 0.93. If pH = 8.93, is $[H^+]$ 1.18×10^{-9} or 8.5×10^9 or some other quantity?

8.5. USING LOGARITHMS TO FIND POWERS AND ROOTS

Until quite recently, calculators and personal computers were not available, and people used logarithms extensively to simplify calculations. Slide rules provided a mechanical method for working with logs, since they have scales marked in proportion to the logarithms of the numbers. Once calculators were readily available that could rapidly multiply, divide, raise to powers, and find roots, use of both slide rules and logarithms for calculations was nearly abandoned. Nonetheless, use of logarithms to find powers and roots may be very convenient. It is essential if you do not have a calculator with a y^x function.

The calculation of powers and roots uses the relationships

$$\log x^n = n \log x$$

$$\log x^{1/n} = \frac{1}{n} \log x$$

These can be combined, so that a number like $759^{2/5}$ (the exponent tells you to raise the number to the second power and find the fifth root of the result) can be found as rapidly as any simpler power or root.

The procedure for each calculation follows the same pattern: (1) Find the logarithm of the number x. (2) Multiply or divide the logarithm as indicated. (3) The result is the logarithm of the required answer. Find the antilog.

EXAMPLE 9

Use logs to calculate $(7.00)^4$.

$$\begin{aligned} \log (7.00)^4 &= 4 (\log 7.00) \\ &= 4 (0.8451) \\ &= 3.3804 \\ (7.00)^4 = \text{antilog } 3.3804 &= 2.40 \times 10^3 \end{aligned}$$

■

EXAMPLE 10

Use logs to calculate $\sqrt[5]{75}$.

$$\begin{aligned} \log (75)^{1/5} &= \frac{1}{5} (\log 75) \\ &= \frac{1}{5} (1.8751) = 0.3750 \\ \sqrt[5]{75} &= \text{antilog } 0.3750 = 2.37 \end{aligned}$$

■

EXAMPLE 11

Use logs to calculate $(759)^{2/5}$.

$$\log (759)^{2/5} = \frac{2}{5} (\log 759)$$

$$= \frac{2}{5} (2.8802) = 1.1521$$

$$(759)^{2/5} = \text{antilog } 1.1521 = 1.42 \times 10^1 = 14.2$$

PROBLEMS

8.13 Use logs to raise to the power.

*(a) $(52)^2$ (b) $(2.9)^3$
(c) $(4)^6$ (d) $(2.0)^5$
(e) $(0.030)^3$ (f) $(0.059)^4$

8.14 Use logs to find the indicated roots.

*(a) $\sqrt[3]{5300}$ (b) $\sqrt[3]{5.0}$
(c) $\sqrt[4]{92}$ (d) $\sqrt[2]{120}$
(e) $(7.0)^{5/2}$ (f) $(25)^{1/2}$
(g) $(50)^{1/4}$

SOLUTIONS TO STARRED PROBLEMS

8.1(a) The logarithm is the exponent. Then

$$\log 10^{-2} = -2$$

The entire exponent, including the minus sign, is the log.

8.2(a) The log of the product is the sum of the logs of the factors.

$$\log 3.00 \times 10^9 = \log 3.00 + \log 10^9$$

The log of 10^9 is simply the exponent 9. The log of 3.0 must be found from the table. Look down the left side to the 30. Then look in the column under 0 for the log of 3.00. The table gives this as 4771. However, the mantissas in the table must always be preceded by a decimal point. Therefore,

$$\log 3.00 = 0.4771$$

$$\log 3.00 \times 10^9 = 0.4771 + 9 = 9.4771$$

(e) To find the log of 466, first rewrite the number in exponential notation. Then find the log.

$$\log 466 = \log 4.66 \times 10^2 = \log 4.66 + \log 10^2$$

The log of 10^2 is 2. To look up the log of 4.66 in the table, look down the left side to the 46 then across to the column under 6. The log is given as 6684. With the decimal, it is 0.6684:

$$\begin{aligned} \log 4.6 \times 10^2 &= \log 4.66 + \log 10^2 \\ &= 0.6684 + 2 \\ &= 2.6684 \end{aligned}$$

8.3(a) $\log 3.0 \times 10^{-9} = \log 3.0 + \log 10^{-9}$

$$\begin{aligned} &= 0.48 + (-9) \\ &= -8.52 \end{aligned}$$

(f) First write the number in exponential notation. Then you are ready to find the log.

$$\begin{aligned} \log 0.0032 &= \log 3.2 \times 10^{-3} \\ &= \log 3.2 + \log 10^{-3} \\ &= 0.51 + (-3) \\ &= -2.49 \end{aligned}$$

8.4(a) No. If a number is between 1 and 10, its log must be between 0 and 1. The characteristic 2 means that the logarithm could correspond to a number like 3.42×10^2, but not to 3.42.

8.8(a) The antilog is the number whose log is given. When the log is a whole number, it is simply written as the exponent of 10.

$$\text{antilog } 10 = 10^{10}$$

(c) The number whose log is 6.8035 is not a whole number power of 10 but the product of a coefficient and a power of 10. The log has two parts: the characteristic, 6, tells which power of 10 is used, 10^6; the mantissa, after the decimal, is the log of the coefficient. Look in Table 8.2 and find the digits 8035. Then look to the left of that row for the first digits 63. Look to the head of

the column for the third digit, 5. The number is then 6.35, with one digit before the decimal.

$$\text{antilog } 6.8035 = \text{antilog } 6 \times \text{antilog } 0.80$$
$$= 6.35 \times 10^6$$

8.9(a) To find the number whose log is -6.29, you must first rewrite the log so that it has a positive mantissa. To do this, add 1.00 to the mantissa and subtract 1 from the characteristic.

$$\begin{array}{rr} -6 & -0.29 \\ \underline{-1} & \underline{+1.00} \\ -7 & +0.71 \end{array}$$

The number whose log is -7 is 10^{-7}. The number whose log is 0.71 is 5.1 from the table. The antilog of -6.29 is then 5.1×10^{-7}.

8.10(a) The pH is defined as $-\log [H^+]$. Therefore, to find the pH, find the log of $[H^+]$ and change the sign.

$$\log [H^+] = \log 10^{-3} = -3$$
$$pH = -(-3) = 3$$

(f) As in part (a),

$$\log [H^+] = \log 0.3 = \log 3 + \log 10^{-1}$$
$$= 0.48 - 1 = -0.52$$
$$pH = -(-0.52) = 0.52$$

8.11(a) Since $pH = -\log [H^+]$.

$$\log [H^+] = -pH = -5$$
$$[H^+] = 10^{-5}$$

(d) $pH = 3.2$. Therefore, $\log [H^+] = -3.2$.

$$\text{antilog} - 3.2 = \text{antilog } (-4 + 0.8)$$
$$= 6.3 \times 10^{-4} = [H^+]$$

8.12(c) $\log [H^+] = -4 + (\text{mantissa})$, so it is between -3 and -4. Therefore, pH is between 3 and 4. The answer 3.38 could be correct, but not the answer 4.38.

8.13(a) To raise to a power, multiply the log by the exponent.

$$\log 52 = \log 5.2 \times 10^1 = 1.72$$
$$\log (52)^2 = 2 (\log 52)$$
$$= 2 (1.72) = 3.44$$
$$(52)^2 = \text{antilog } 3.44 = 2.7 \times 10^3$$

8.14(a) To find the root, divide the log by the index.

$$\log 5300 = \log 5.3 \times 10^3 = 3.72$$
$$\log (5300)^{1/3} = \frac{1}{3} (\log 5300)$$
$$= \frac{1}{3} (3.72) = 1.24$$

The required root is the antilog of 1.24, 1.7×10^1 (or 17).

CHAPTER

9

EQUATIONS II

9.1. DERIVING EQUATIONS

Many times, you cannot find the exact equation you need for a calculation, but you can find other equations relating to the system under consideration. It is possible to derive a useful equation from other equations by applying some simple mathematical rules.

1. Things equal to the same thing are equal to each other. That is,

$$\text{if} \quad a = c \quad \text{and} \quad b = c, \quad \text{then} \quad a = b$$

   This is the basis of the next two methods, but it is convenient to consider them separately.

2. Since the two sides of an equation are equal (mathematically, are different names for the same thing), the two sides of one equation can be used in a variety of operations on the two sides of another equation.

   If

$$a = b \quad \text{and} \quad c = d$$

   then

$ac = bd$	An equation remains true if both sides are multiplied by the same quantity.
$\dfrac{a}{c} = \dfrac{b}{d}$	Both sides can be divided by the same quantity.

$$a + c = b + d$$
$$a - c = b - d$$
$$c - a = d - b$$

An equation remains true if the same quantity is added to or subtracted from both sides.

3. A mathematical expression may be substituted for its equivalent in an equation. That is, if $c = de$, then de can be substituted for c:

$$a = bc$$

$$a = bde$$

EXAMPLE 1

Boyle's law states that, for a given sample of a gas at constant temperature, the product of the pressure and volume is constant:

$$PV = k$$

This means that any change in the pressure produces a change in the volume that will keep their product constant. When the same unit is measured in several different experiments, or under several different conditions, it is convenient to use the same letter for the quantity each time, but use a subscript to designate which experimental measurement is meant. Using this notation, P_1 is the pressure in experiment 1; P_2 is the pressure in experiment 2.

$P_1V_1 = K$ for the first set of measurements

$P_2V_2 = K$ for the second set of measurements

Boyle's law states that K is the same for both. By the rule that things equal to the same thing are equal to each other:

$$P_1V_1 = P_2V_2$$

This now is an equation that can be solved for any one of the four quantities if the other three are known. For instance, if you measure the pressure and volume under one set of conditions, you can find what pressure would give a specified volume or what volume would be occupied by the gas sample at a given pressure. ■

EXAMPLE 2

Equations comparing the rates of movement of molecules of different gases can be developed using either the first or the second method. The kinetic energy (energy of motion) of a gas is related to the temperature,

so that the kinetic energy is the same for any gas at a given temperature. Kinetic energy, KE, is defined by the equation

$$\mathrm{KE} = \frac{1}{2} mv^2$$

where m is the mass of the particles, in this case the molecules (the molecular weight), and v is the average velocity (essentially speed).

Using method 1, we write the equations for two gases

$$\mathrm{KE}_1 = \frac{1}{2} m_1 v_1^2$$

$$\mathrm{KE}_2 = \frac{1}{2} m_2 v_2^2$$

At the same temperature,

$$\mathrm{KE}_1 = \mathrm{KE}_2$$

Therefore,

$$\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_2 v_2^2$$

This equation can be rearranged in various ways. For example, we might wish to compare the rates of movement, the velocities. Canceling the 1/2's and rearranging to give the ratio of the velocities gives the equation

$$\frac{v_1^2}{v_2^2} = \frac{m_2}{m_1}$$

or

$$\frac{v_1}{v_2} = \sqrt{\frac{m_2}{m_1}}$$

This is *Graham's law of diffusion*, which relates the relative speed at which two molecules move (measured perhaps by how fast a gas escapes through a small opening) to the relative molecular weights.

The second method could equally well have been used. If the ratio of the velocities is wanted, write equations for the velocities; then take their ratio.

$$\mathrm{KE} = \frac{1}{2} mv^2$$

Solving for v, we obtain

$$v^2 = \frac{2\text{KE}}{m}$$

$$v = \sqrt{\frac{2\text{KE}}{m}}$$

and for two different gases.

$$v_1 = \sqrt{\frac{2\text{KE}_1}{m_1}}$$

$$v_2 = \sqrt{\frac{2\text{KE}_2}{m_2}}$$

The ratio of velocities is found by dividing one equation by the other.

$$\frac{v_1}{v_2} = \frac{\sqrt{\dfrac{2\text{KE}_1}{m_1}}}{\sqrt{\dfrac{2\text{KE}_2}{m_2}}} = \frac{\sqrt{\dfrac{\cancel{2}\text{KE}_1}{m_1}}}{\sqrt{\dfrac{\cancel{2}\text{KE}_2}{m_2}}}$$

$$\frac{v_1}{v_2} = \sqrt{\frac{m_2}{m_1}\,\frac{\text{KE}_1}{\text{KE}_2}}$$

If the gases are at the same temperature, $\text{KE}_1 = \text{KE}_2$, and they can be canceled. Then

$$\frac{v_1}{v_2} = \sqrt{\frac{m_2}{m_1}}$$ ■

EXAMPLE 3

Use Coulomb's law to derive an equation comparing the forces between charged particles under different conditions. The equation for Coulomb's law is

$$F = K\,\frac{q_1 q_2}{r^2}$$

Since the subscripts 1 and 2 are already in use here, another notation will have to be used. Let us indicate the second conditions with a prime so that the forces under the two conditions are shown as F and F' (read "F prime").

$$F' = K'\,\frac{q_1' q_2'}{(r')^2}$$

To find the ratio of force under two conditions, divide F by F'. The right sides of the equation must be divided at the same time, producing a new equation.

$$\frac{F}{F'} = \frac{\dfrac{Kq_1q_2}{r^2}}{\dfrac{K'q_1'q_2'}{(r')^2}} = \frac{q_1q_2K\,(r')^2}{q_1'q_2'K'r^2}$$

Note that F' is still directly proportional to q_1' when they are both in the denominator on opposite sides of the equation. It is inversely proportional to the square of r', since one of them is in the numerator and the other in the denominator.

This general equation can be simplified for various special cases. For instance, if the surroundings are the same in both instances, $K = K'$ and they can be canceled. ■

A very common use of method 2 in chemistry is the calculation of the concentrations of substances present in equilibrium mixtures. (In the following calculations the concentration of a species in solution is indicated by writing the formula for the molecule or ion in square brackets. For example, $[H^+]$ is read "the hydrogen ion concentration" and is used as a variable just as if it were a single letter.) Only certain types of equilibrium constants are given in tables; others must be derived from them if needed.

■ EXAMPLE 4

Derive an equation for the hydrolysis constant, K_h, for HCN,

$$K_h = \frac{[HCN][OH^-]}{[CN^-]}$$

using the known values of K_a and K_w.

$$K_a = \frac{[CN^-][H^+]}{[HCN]} = 5 \times 10^{-10}$$

$$K_w = [H^+][OH^-] = 1 \times 10^{-14}$$

Neither of these is exactly what is needed, but each has some of the terms wanted. Both contain $[H^+]$, which is not wanted, but it can be canceled out if one equation is divided by the other. Which then should be divided by which?

The expression for K_w has $[OH^-]$ in the numerator, where it is wanted, but the expression for K_a has [HCN] in the denominator when it is needed in the numerator, and $[CN^-]$ in the numerator when it is needed in the denominator. Clearly, then, the expression for K_a would have to be inverted to put the [HCN] and $[CN^-]$ in the correct positions. Inverting the expression for K_a gives

$$\frac{1}{K_a} = \frac{[HCN]}{[H^+][CN^-]}$$

Multiplying this by the expression for K_w would give an equation with all the terms in the right place.

$$K_w \frac{1}{K_a} = [H^+][OH^-] \frac{[HCN]}{[H^+][CN^-]} = \frac{[OH^-][HCN]}{[CN^-]}$$

and

$$K_w \frac{1}{K_a} = 1 \times 10^{-14} \times \frac{1}{5 \times 10^{-10}} = 2 \times 10^{-5}$$

Since multiplying by $1/K_a$ is the same as dividing by K_a, the procedure is usually described as dividing K_w by K_a.

$$\frac{K_w}{K_a} = \frac{[H^+][OH^-]}{\frac{[H^+][CN^-]}{[HCN]}} = \frac{1 \times 10^{-14}}{5 \times 10^{-10}}$$

$$K_h = \frac{[HCN][OH^-]}{[CN^-]} = 2 \times 10^{-5}$$ ■

The third approach to deriving equations from other equations, substitution of an expression for its equivalent, is enormously useful. Sometimes you need to make a calculation using quantities that can be measured conveniently, but the original equation does not contain terms for your measured quantities. Sometimes you are interested in seeing how a given quantity will be affected by various kinds of variables. For both purposes it is useful to derive an equation for one quantity *in terms of* certain specified other quantities. This means that the equation for the one quantity contains terms for the specified quantities. There may be other terms present also, of course, and the specified quantities may appear in the denominator of a fraction, in a squared term, or wherever they are needed to make the equation correct.

■ EXAMPLE 5

The energy of a photon of light is proportional to the frequency of the light, ν. The proportionality constant, called *Planck's constant*, is always symbolized by h. (It has a value of 6.625×10^{-16} J-sec.)

$$E = h\nu \qquad \text{where } h = \text{Planck's constant and } \nu = \text{frequency}$$

The frequency of light is defined by the equation

$$\nu = \frac{c}{\lambda} \qquad \text{where } c = \text{speed of light and } \lambda = \text{wavelength}$$

(ν and λ are the Greek letters nu and lambda.)

Derive an equation for the energy of a photon of light of a given wavelength.

Substitute c/λ for its equivalent:

$$E = h\nu = h\frac{c}{\lambda}.$$

■

■ EXAMPLE 6

You wish to calculate the molecular weight, M, from measurements of P, V, and T for a weighed sample of a gas. Write an equation for M in terms of P, V, and T, starting with the equations

$$PV = nRT \qquad \text{and} \qquad n = \frac{w}{M}$$

Any of several procedures could be used. One way is to start by solving both equations for the one factor that appears in both, n.

$$n = \frac{PV}{RT} \qquad \text{and} \qquad n = \frac{w}{M}$$

Then, since things equal to the same thing are equal to each other,

$$\frac{PV}{RT} = \frac{w}{M}$$

Solve for M:

$$M = \frac{wRT}{PV}$$

Another method would have been to substitute the value of n into the first equation.

$$PV = \frac{w}{M} RT$$

Solve for M:

$$M = \frac{wRT}{PV}$$ ■

PROBLEMS

9.1 The pressure P at a depth h under a liquid of density d is given by

$$P = hdg \qquad g = \text{acceleration due to gravity}$$

The force F on an area A under a pressure P is

$$F = PA$$

*(a) Give an equation for force in terms of h and d (and any other quantities needed).
*(b) Give an equation for density of a liquid in terms of P, h, and g.

9.2 Given the equations

$$PV = nRT \qquad \text{and} \qquad n = \frac{w}{M} \qquad (R \text{ is a constant})$$

*(a) Write an equation for P_1/P_2.
(b) Write an equation for T_1/T_2.
(c) Write an equation for w in terms of P, V, and T.

9.3 If a ball is thrown upward at initial velocity v, the time to reach the greatest height is

$$t = \frac{v}{g} \qquad g = \text{acceleration due to gravity}$$

The height h is given by

$$h = \frac{1}{2} gt^2$$

(a) Write an equation for h that contains v.
(b) The potential energy, PE, possessed by the ball at the highest point is

$$\mathrm{PE} = mgh$$

Use the original equation for h to derive an equation for PE in terms of t.
(c) Use the equation derived in part (a) to derive an equation for PE in terms of v. Compare your answer with the kinetic energy of the ball when it was first thrown.

$$\mathrm{KE} = \frac{1}{2} mv^2$$

9.4 If

$$PV = \frac{1}{3} Nmv^2$$

$$PV = \frac{2}{3} \mathrm{KE}$$

$$PV = RT$$

write equations for KE that contain
*(a) v (b) T

9.5 If

$$[\mathrm{Ag}^+][\mathrm{Cl}^-] = K_{sp}$$

and

$$\frac{[\mathrm{Ag}^+][\mathrm{NH}_3]^2}{[\mathrm{Ag}\,(\mathrm{NH}_3)_2^+]} = K_i$$

write an equation for

$$\frac{[\mathrm{Cl}^-][\mathrm{Ag}\,(\mathrm{NH}_3)_2^+]}{[\mathrm{NH}_3]^2}$$

9.6 If in a solution

$$[Ni^{2+}][S^{2-}] = 1.4 \times 10^{-24}$$

and

$$[Cd^{2+}][S^{2-}] = 3.6 \times 10^{-29}$$

what is the ratio of $[Ni^{2+}]$ to $[Cd^{2+}]$?

9.2. SOLVING SIMULTANEOUS EQUATIONS

There are many situations in which there are two or more unknown quantities. Equations describing such situations can be solved if there are as many independent equations as there are unknowns. Equations are independent if they describe different information, but not if one equation is merely a rearrangement of another. Since the relationships described by the different equations are all true at the same time, such equations are called *simultaneous equations.*

There are several methods of solving simultaneous equations. You can draw graphs of both equations; the points where the graphs intersect (points that are common to both graphs) are solutions to the equations. There are computer methods for solving complicated sets of simultaneous equations. However, the simultaneous equations encountered in a beginning chemistry course can usually be solved by one of two simple methods.

Both methods combine the equations to obtain a new equation that contains only one unknown. This new equation is solved for that unknown. Then the value of that unknown is substituted into one of the original equations, which can then be solved for the value of the other unknown.

The first method is to add, subtract, multiply, or divide the left sides and the right sides of the equations to obtain a new equation in one unknown. The equations

$$x + y = 3$$

$$x - y = 27$$

can be added to give

$$x + x + y - y = 3 + 27$$

Combining terms gives

$$2x = 30$$

$$x = 15$$

Substituting this value of x into the first equation gives

$$15 + y = 3$$

$$y = -12$$

Alternatively, the second equation could have been subtracted from the first to give

$$x + y - x - (-y) = 3 - 27$$

$$2y = -24$$

$$y = -12$$

Substituting this value into either of the original equations gives the value $x = 15$.

If x and y had originally been multiplied rather than added, one equation could have been divided by the other.

■ EXAMPLE 7

The compounds $BaSO_4$ and $BaCO_3$ are quite insoluble in water; the equations for the maximum concentration of the ions in solution are

$$[Ba^{2+}][SO_4^{2-}] = 1 \times 10^{-10}$$

$$[Ba^{2+}][CO_3^{2-}] = 8 \times 10^{-9}$$

where $[Ba^{2+}]$ means the concentration of Ba^{2+} ions in solution. A mixture of solid $BaSO_4$ and solid $BaCO_3$ is in equilibrium with a solution containing SO_4^{2-} and CO_3^{2-}. (This statement simply means that the equations apply simultaneously to the situation.) If $[SO_4^{2-}] = 10^{-2}$, what is $[CO_3^{2-}]$?

Divide one equation by the other. This will eliminate $[Ba^{2+}]$. Both sides of the equations must be divided.

$$\frac{[Ba^{2+}][CO_3^{2-}]}{[Ba^{2+}][SO_4^{2-}]} = \frac{8 \times 10^{-9}}{1 \times 10^{-10}}$$

$$\frac{[CO_3^{2-}]}{[SO_4^{2-}]} = 8 \times 10^{1}$$

This ratio is a useful number even if you did not have the actual value of either $[CO_3^{2-}]$ or $[SO_4^{2-}]$. For this problem, you do have the value of the sulfate concentration, $[SO_4^{2-}]$, and can substitute this into the equation.

$$\frac{[CO_3^{2-}]}{10^{-2}} = 8 \times 10^{1}$$

$$[CO_3^{2-}] = 8 \times 10^{-1}$$

■

If the calculation in Example 7 looked complicated, you may have been confused by the chemical notation. The calculation was essentially

$$ax = 1 \times 10^{-10}$$

$$ay = 8 \times 10^{-9}$$

If $x = 10^{-2}$, what is y?

There are two unknowns, a and y. Dividing one equation by the other eliminates a.

$$\frac{\cancel{a}y}{\cancel{a}x} = \frac{8 \times 10^{-9}}{1 \times 10^{-10}}$$

$$\frac{y}{x} = 8 \times 10^{1}$$

Substituting the value of x, we obtain

$$\frac{y}{10^{-2}} = 8 \times 10^{1}$$

$$y = 8 \times 10^{-1}$$

The calculation is the same regardless of the notation used for the unknowns. The advantage of using the chemical terms is that they remind you of the actual situation being considered.

The second method for solving simultaneous equations is to solve one equation for one unknown in terms of the other. Then this quantity is substituted into the second equation. If both equations are linear and fairly simple, you can start with either one and solve for either of the two unknowns. When one equation is complicated or contains terms raised to higher powers, it is easier to solve the simpler equation for one unknown in terms of the other, and then to substitute this value into the more complicated equation.

EXAMPLE 8

Solve the simultaneous equations for x and y.

$$\frac{2x}{y} = 6 \qquad \text{and} \qquad x + y = 12$$

The second equation is simpler, so solve it for one of the unknowns.

$$\begin{aligned} x + y &= 12 \\ x &= 12 - y \end{aligned}$$

Substitute this value of x into the first equation and solve for y.

$$\begin{aligned} \frac{2x}{y} &= 6 \\ \frac{2\,(12 - y)}{y} &= 6 \\ 2\,(12 - y) &= 6y \\ 24 - 2y &= 6y \\ 24 &= 8y \\ 3 &= y \end{aligned}$$

Now substitute this value of y into either of the original equations and solve for x.

$$\begin{aligned} x + y &= 12 \\ x + 3 &= 12 \\ x &= 9 \end{aligned}$$

Check the answers by substituting into the other equation.

$$\begin{aligned} \frac{2x}{y} &= 6 \\ \frac{2\,(9)}{3} &= 6 \\ 6 &= 6 \end{aligned}$$

EXAMPLE 9

One method of analyzing an aluminum–nickel alloy is to dissolve it in acid and measure the hydrogen produced. If a sample of alloy weighing 2.00 g produced 1176 mL of hydrogen at standard temperature and pressure, the following pair of simultaneous equations can be written to describe the experiment.

Let

a = grams of aluminum and n = grams of nickel

Then

$$a + n = 2.00$$

This equation says that the grams of aluminum plus the grams of nickel equal the total grams of the alloy:

$$\frac{a}{9.00} + \frac{n}{29.4} = \frac{1176}{11{,}200}$$

This equation relates the amounts of aluminum and of nickel to the amount of hydrogen produced. (See your textbook for further explanation. The numbers ought to be written with their correct units. Here the units were omitted to focus attention on the mathematical operations.)

Since the two equations convey different information, they are independent and can be solved for the two unknowns. Clearly, it would be simpler to solve the first than the second.

$$a + n = 2.00$$
$$a = 2.00 - n$$

Substitute this value of a into the second equation:

$$\frac{2.00 - n}{9.00} + \frac{n}{29.4} = \frac{1176}{11{,}200}$$

To save arithmetic, multiply the equation by the denominators of the fractions that contain the unknown, n, but not by the 11,200.

$$29.4\,(2.00 - n) + 9.00n = \frac{1176}{11{,}200}\,(9.00)(29.4)$$
$$58.8 - 29.4n + 9.00n = 27.7$$
$$31.1 = 20.4n$$
$$1.52 = n$$

Then

$$a + n = 2.00$$
$$a + 1.52 = 2.00$$
$$a = 0.48$$

There are 0.48 g of aluminum and 1.52 g of nickel in the alloy.

In a real situation there may be physical limitations on the answers. For instance, if you had found (erroneously) that the value of n (the mass of nickel) was 4.5 g, then solving for a would have given $a = -2.5$ g. These answers might satisfy the equation but could not possibly be correct. Since n represents the mass of nickel, the value of n could not be greater than the mass of the original sample of alloy. Since a represents the mass of aluminum, the value of a could not be negative. ■

Students sometimes are so pleased at having solved an equation and found an answer that they omit two steps that should be included. **Check your answer to see whether it makes sense. Reread the original problem, to make sure that the answer is the one needed.**

PROBLEMS

9.7 Solve for the unknowns.

*(a) $x + y = 9$, $xy = 20$

(b) $x - y = 5.0$, $xy = 66$

(c) $x - y = -0.50$, $xy = 33$

*(d) $x + y = 12$, $\dfrac{x}{y} = 3$

(e) $x + y = 2$, $\dfrac{x}{y} = -7$

(f) $x + y = 9$, $\dfrac{x}{2} + \dfrac{y}{5} = 3$

9.8 *(a) $ab = 10$
$ac = 20$
What is c if $b = 5$?

(b) $de = 9 \times 10^{-7}$
$df = 6 \times 10^{-3}$
What is e if $f = 2 \times 10^{-2}$?

*(c) $$\frac{[H^+]^2[S^{2-}]}{10^{-1}} = 1.2 \times 10^{-20}$$

$[Ni^{2+}][S^{2-}] = 1.4 \times 10^{-24}$
What is $[Ni^{2+}]$ when $[H^+] = 1.0$?

(d) Using the equations of part (c), what is $[Ni^{2+}]$ when $[H^+] = 1.0 \times 10^{-7}$?

(e) $[Cd^{2+}][OH^-]^2 = 1.2 \times 10^{-14}$

$[H^+][OH^-] = 1.0 \times 10^{-14}$

What is $[Cd^{2+}]$ if $[H^+] = 1.0 \times 10^{-7}$?

9.3. SOLVING HIGHER-ORDER EQUATIONS

Equations in which the unknown appears to higher than the first power (is squared, cubed, etc.) are very common. The approach to solving such equations depends on the type of equation. Two methods are given in the paragraphs that follows and another is given in Section 9.4.

Whatever method is used to solve a higher-order equation, there will be as many roots as the highest power of the unknown. That is, if the equation contains x^3, there will be three roots. If the equation contains x^4, there will be four roots. (An equation can sometimes appear to have only one root, because all roots are identical.) All roots are equally true mathematically, so all should be given as solutions. When the equation describes a physical situation, there is sometimes a reason to prefer one of the roots to another. For example, a substance cannot have a negative mass.

9.3.A. Equations in Which the Unknown Appears to Only One Power

Some equations have the unknown present to the same power in all terms in which it appears. If it appears as x^3, there are no terms containing x^2 or x.

Equations in which the unknown appears to a single power (or root) can be solved by the same principles used for linear equations. That is, consider what operation has been performed, and then perform the inverse *on both sides of the equation*. For example, if the equation contains x^4, the inverse operation would be to take the fourth root of both sides. If the equation contains the cube root of x, the inverse operation would be to cube both sides.

If the variable appears to an even power in the equation, the root could be either positive or negative, since raising either one to an even power would give an even number. If the variable appears to an odd power, the sign of the root is the same as the sign of the original number. That is, the cube root of a positive number is positive, and the cube root of a negative number is negative. All roots of such an equation are identical.

■ **EXAMPLE 10**

Solve for x:

$$x^2 = 25$$

Take the square root of both sides.

$$\sqrt{x^2} = \sqrt{25}$$
$$x = \pm 5$$

For even powers of x, it is necessary to indicate that the root may be positive or negative, since raising either to an even power would give a positive number. There is no way to tell from the equation whether one of the two roots, $+5$ or -5, is to be preferred. ■

■ **EXAMPLE 11**

Solve for x.

$$x^5 = 72.0$$
$$x = \sqrt[5]{72.0}$$

The fifth root can be found using logarithms (Chapter 8) or using the appropriate function on your calculator.

$$x = 2.35$$

The root must be positive; raising a negative number to an odd power would give a negative number. ■

■ **EXAMPLE 12**

Solve for x.

$$x^2 = 4y - 12$$
$$\sqrt{x^2} = \sqrt{4y - 12}$$
$$x = \pm\sqrt{4y - 12} = \pm\sqrt{4(y - 3)}$$
$$= \pm\sqrt{4}\sqrt{y - 3} = \pm 2\sqrt{y - 3}$$

Although it is not necessary to factor out the 4 in this fashion, it is sometimes useful. Further, you should know that it can be done so that you will understand what is happening if you see someone else solve an equation in this way. ■

EXAMPLE 13

Solve for x.

$$\sqrt{x} = 3m + 1$$

Here, it is necessary to square both sides of the equation to obtain x to the first power.

$$(\sqrt{x})^2 = (3m + 1)^2$$
$$x = (3m + 1)(3m + 1)$$
$$= 3m\,(3m + 1) + 1\,(3m + 1)$$
$$= 9m^2 + 3m + 3m + 1$$
$$x = 9m^2 + 6m + 1$$

The entire right side must be squared, not just one of the terms. ■

PROBLEMS

9.9 Solve for the unknown. ***Note:*** A quantity like $[H^+]$ can only have a positive value, since $[H^+]$ stands for the concentration of hydrogen ion.

(a) $x^2 = 36$ (b) $x^2 = 10^{-2}$
*(c) $[H^+]^2 = 10^{-2}$ (d) $[OH^-]^2 = 10^{-8}$
(e) $[I^-]^3 = 10^{-6}$ *(f) $y^3 + 8 = 0$
(g) $z^2 = 1.44$ (h) $[Ag^+]^2\,(10^{-21}) = 10^{-51}$

9.3.B. Solving Quadratic Equations

Quadratic equations, equations in which the highest power of x is the second power, can be solved by use of the quadratic formula. To do this, start by rearranging the equation so that all terms are on the same side. The equation should then have the form

$$ax^2 + bx + c = 0$$

Identify the numerical values, with their + or − signs, of the coefficient of the x^2 term, a, the coefficient of the x term, b, and the term that

does not contain x, c. Substitute these values into the quadratic formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

and solve for x.

There will be two roots, that is, two values for x, one arising from the positive value of the square-root quantity and one from the negative value. This is in accordance with the general rule that the number of roots of an equation is equal to the order of the equation, that is, the highest power of the unknown.

EXAMPLE 14

Solve for x.

$$x^2 = 5x + 14$$

First, move all terms to the same side to put the equation into the required form. Here, that requires subtracting $(5x + 14)$ from both sides.

$$x^2 - 5x - 14 = 0$$

The coefficient of the x^2 term is 1 (which is not shown as the number but is implied). Therefore, a for the quadratic formula is 1. The coefficient of the x term is -5, which becomes b of the formula. The value of c, the term that does not contain x, is -14. The coefficients include the negative sign. Substitute the values of a, b, and c into the quadratic formula.

$$a = 1 \qquad b = -5 \qquad c = -14$$

$$\begin{aligned} x &= \frac{-(-5) \pm \sqrt{(-5)^2 - 4\,(1)(-14)}}{2\,(1)} \\ &= \frac{5 \pm \sqrt{25 + 56}}{2} \\ &= \frac{5 \pm \sqrt{81}}{2} = \frac{5 \pm 9}{2} \end{aligned}$$

The two roots are

$$x = \frac{5 + 9}{2} = 7 \qquad \text{and} \qquad x = \frac{5 - 9}{2} = -2$$

EXAMPLE 15

Solve $2y^2 - 3y = 0$ for y.

This can be solved using the quadratic formula:

$$a = 2 \qquad b = -3 \qquad c = 0$$

$$y = \frac{-(-3) \pm \sqrt{(-3)^2 - 4\,(2)(0)}}{2\,(2)}$$

$$y = \frac{3 + \sqrt{9 - 0}}{4} \qquad \text{or} \qquad y = \frac{3 - \sqrt{9 - 0}}{4}$$

$$y = \frac{3}{2} \qquad y = 0$$

However, it is also possible to solve this equation by a shorter method, since there is a common factor, y, present in all terms.

$$2y^2 - 3y = y\,(2y - 3) = 0$$

If the product of two factors is 0, one of the factors must have a value of 0. Therefore, the two roots of the equation can be found by setting each of the factors in turn equal to 0.

$$y = 0 \qquad \text{or} \qquad \begin{aligned} 2y - 3 &= 0 \\ 2y &= 3 \\ y &= \frac{3}{2} \end{aligned}$$

■

PROBLEMS

9.10 Solve for the unknown. There will be two answers for each.

*(a) $x^2 = 5x - 6$
(b) $x^2 = 8x + 9$
(c) $2x^2 + 2x = 30$
(d) $4x = x^2 - 45$
(e) $x^2 - 1.7x + 0.3 = 0$
(f) $x^2 + 3.1x - 1.8 = 0$

9.11* (a) Solve for $[CN^-]$.

$$\frac{[H^+][CN^-]}{[HCN]} = 4.0 \times 10^{-10}$$

$$[HCN] = 1.00 \times 10^{-2} \qquad \text{and} \qquad [CN^-] = [H^+].$$

[*Hint:* If $[CN^-] = x$, what does $[H^+]$ equal?]

(b) Solve the equation in part (a) for $[H^+]$ if

$$[HCN] = 5.0 \times 10^{-3} \qquad \text{and} \qquad [H^+] = [CN^-]$$

9.12 Solve for $[H^+]$.

$$\frac{[H^+][NO_2^-]}{[HNO_2]} = 4.5 \times 10^{-4}$$

$$[NO_2^-] = [H^+];\ [HNO_2] = (1.00 \times 10^{-2} - [H^+])$$

9.4. GRAPHING EQUATIONS

To graph an equation in two variables, assign any convenient values to one of the variables, substitute each value in turn into the equation, and solve for corresponding values of the other variable. It is usually convenient to set up a table of the sets of values. Then label the coordinates appropriately so that they will cover the values used. Locate the points on the graph and connect them with a smooth line. It may be necessary to locate additional intermediate points (such as $x = 1/2$) to be sure of the shape of a curve where it changes sharply.

■ EXAMPLE 16

Draw a graph of the equation $x - y = 2$.

Select values of x and find the corresponding values of y.

x	0	1	2	3	4	5	6
y	−2	−1	0	1	2	3	4

■

Plot the points and draw the graph (Figure 9.1).

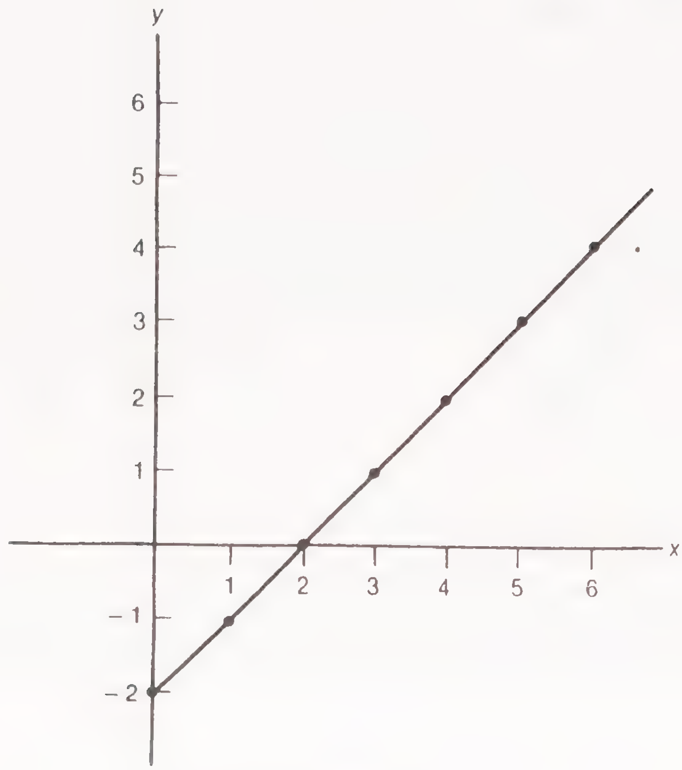


FIGURE 9.1

A graph can be used to find the solution to an equation. If the equation is set up in such a way that $y = 0$, the value(s) of x where it crosses the horizontal axis (where $y = 0$) are solutions to the equation. This is an especially useful technique for equations that are difficult to solve by other methods.

EXAMPLE 17

Draw a graph of the equation $x^2 + x - 6 = 0$. From the graph find what values of x are solutions to the equation, that is, what values of x make the left side of the equation equal to 0, the right side. To do this, write the equation as

$$y = x^2 + x - 6$$

Choose values of x and calculate y. Plot the result on a graph with the values of x on the abscissa and y on the ordinate (Figure 9.2). If necessary, use intermediate or fractional values of x to determine the shape where the shape of the graph changes rapidly. Here, a value of x is

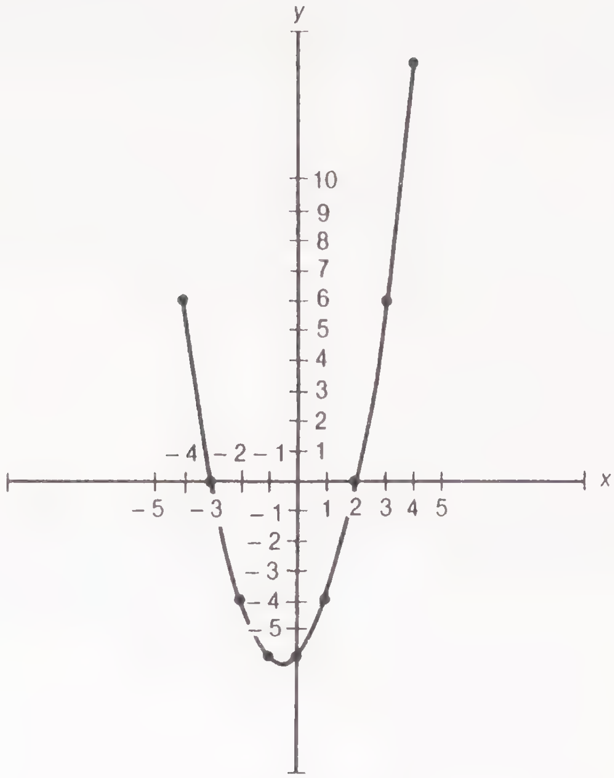


FIGURE 9.2

needed between 0 and −1 in order to determine the shape of the bottom of the curve. Any choice is possible, but $x = -1/2$ is convenient.

x	0	+1	+2	+3	+4	−1	−2	−3	−4	−1/2
y	−6	−4	0	+6	+14	−6	−4	0	+6	−6 1/4

SOLUTIONS: $x = -3$ and $x = +2$. ■

Simultaneous equations can be solved by graphing both equations on the same graph. The point where the lines cross is a point that satisfies both equations and is, therefore, a solution to the set of simultaneous equations.

■ EXAMPLE 18

Use a graph to solve the simultaneous equations.

$$(1) \qquad 2x - y = 8$$

$$(2) \qquad x + 3y = 9$$

Prepare a table of data for each equation, choosing values of one variable and solving for the other, for each point. Plot both equations on the same graph.

Equation (1):

x	0	1	2	3	4	5
y	8	6	4	2	0	-2

Equation (2):

x	9	6	3	0	-3
y	0	1	2	3	4

(If values of x ranging from 0 to 5 had been chosen for equation 2, the values of y would have been fractions. Choosing values of y ranging from 0 to 5 gave numbers that were easier to plot, but either way would have been correct.) (See Figure 9.3.)

SOLUTION: The point (3, 2) is common to both lines, so the solution is $x = 3$, $y = 2$. ■

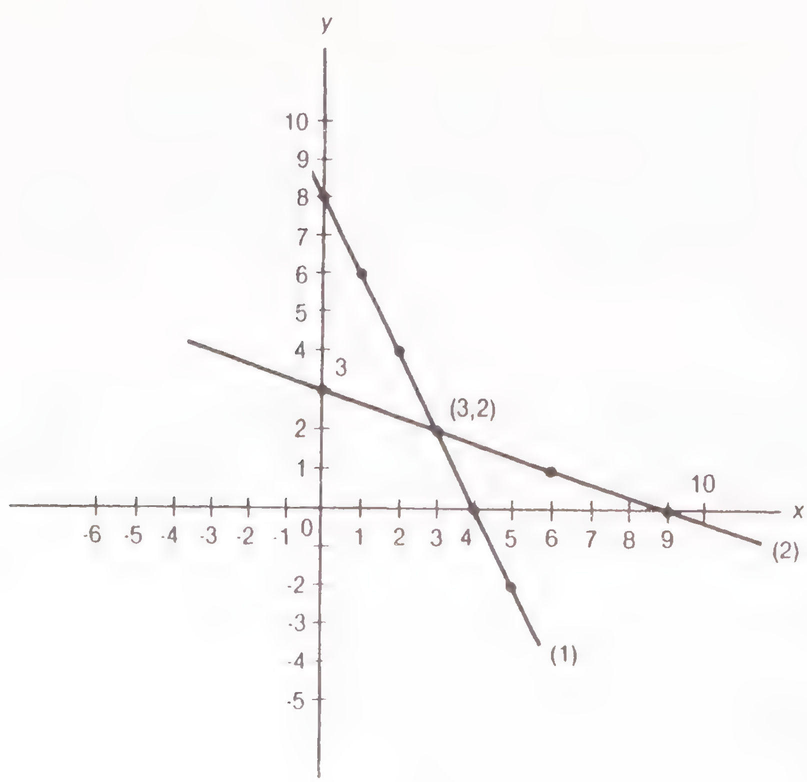


FIGURE 9.3

PROBLEM

9.13 Graph the following equations.

(a) $x + y = 10$ (b) $2x - y = 0$

(c) $x^2 = 2y$ (d) $y = x^2 - 5x + 3$

(e) $x^2 + y^2 = 9$

SOLUTIONS TO STARRED PROBLEMS

9.1(a) The equation given for force is $F = PA$. To obtain an equation that contains h and d, look for some equation that expresses a relationship between P (and/or A) and the needed quantities. The equation is given.

$$P = hdg$$

Substituting this value for P into the equation for force gives

$$F = hdgA$$

(b) There is an equation already given that contains all the necessary units: d, P, h, and g. All that must be done is to solve the equation for density, d.

$$P = hdg$$

$$d = \frac{P}{hg}$$

9.2(a) To write an equation for the ratio of pressures under two sets of conditions, write the original equation for the two sets of conditions and then divide.

$$P_1V_1 = n_1RT_1$$
$$P_2V_2 = n_2RT_2$$

Since R is constant, it is the same both times.

Upon dividing, we obtain

$$\frac{P_1V_1}{P_2V_2} = \frac{n_1\cancel{R}T_1}{n_2\cancel{R}T_2}$$

To solve for P_1/P_2, multiply both sides by V_2/V_1.

$$\frac{P_1}{P_2} = \frac{n_1 T_1 V_2}{n_2 T_2 V_1}$$

9.4(a) The easiest way to solve this problem is to use the rule that things equal to the same thing are equal to each other. An equation that contains KE is $PV = 2/3\text{KE}$. An equation that contains v is $PV = 1/3Nmv^2$. Setting the two quantities that equal PV equal to each other, we obtain

$$\frac{2}{3}\text{KE} = \frac{1}{3}Nmv^2$$

$$\text{KE} = \frac{1}{2}Nmv^2$$

9.7(a) Solve the first equation for x in terms of y.

$$x + y = 9$$
$$x = 9 - y$$

Then substitute this value of x into the other equation.

$$xy = 20$$
$$y(9 - y) = 20$$
$$9y - y^2 = 20 \text{ rearrange}$$
$$y^2 - 9y + 20 = 0$$

Now, by factoring or by the quadratic formula, $y = 4$ or 5. Then

$$x = 9 - y \qquad \text{or} \qquad x = 9 - x$$
$$x = 9 - 4 = 5 \qquad\qquad x = 9 - 5 = 4$$

The two sets of roots are

$$x = 5, \quad y = 4 \qquad \text{and} \qquad x = 4, \quad y = 5$$

(d) Solve the first equation for x in terms of y.

$$x + y = 12$$
$$x = 12 - y$$

Substitute this value of x into the other equation.

$$\frac{x}{y} = 3$$

$$\frac{12 - y}{y} = 3$$

$$12 - y = 3y$$

$$12 = 4y$$

$$3 = y$$

Then find x.

$$x = 12 - y = 12 - 3 = 9$$

9.8(a) This problem can be done in either of two ways. In some problems one is easier and in some the other is easier. Here it does not matter.

METHOD 1: Substitute the value of b and see what can be solved.

$$ab = 10 \qquad b = 5$$

$$a\,(5) = 10$$

$$a = 2$$

Now substitute this value of a into the other equation.

$$ac = 20$$

$$(2)\,c = 20$$

$$c = 10$$

METHOD 2: Since you are not interested in a, eliminate it by dividing one equation by the other.

$$\frac{ac}{ab} = \frac{20}{10} \qquad \text{Cancel the } a\text{'s and the 10's.}$$

$$\frac{c}{b} = 2$$

Now substitute the value of b.

$$\frac{c}{5} = 2$$

$$c = 10$$

(c) First simplify the first equation.

$$\frac{[H^+]^2\,[S^{2-}]}{10^{-1}} = 1.2 \times 10^{-20}$$

$$[H^+]^2\,[S^{2-}] = 1.2 \times 10^{-21}$$

Then, use either of the methods of Problem 9.8(a).

$$(1)^2\,[S^{2-}] = 1.2 \times 10^{-21}$$

$$[S^{2-}] = 1.2 \times 10^{-21}$$

$$[Ni^{2+}][S^{2-}] = 1.4 \times 10^{-24}$$

$$[Ni^{2+}](1.2 \times 10^{-21}) = 1.4 \times 10^{-24}$$

$$[Ni^{2+}] = \frac{1.4 \times 10^{-24}}{1.2 \times 10^{-21}} = 1.2 \times 10^{-3}$$

or else divide:

$$\frac{[Ni^{2+}]\cancel{[S^{2-}]}}{[H^+]^2\,\cancel{[S^{2-}]}} = \frac{1.4 \times 10^{-24}}{1.2 \times 10^{-21}} = 1.2 \times 10^{-3}$$

Then substitute the value $[H^+] = 1$.

$$\frac{[Ni^{2+}]}{1^2} = 1.2 \times 10^{-3}$$

$$[Ni^{2+}] = 1.2 \times 10^{-3}$$

9.9(c) Consider $[H^+]$ as a single quantity. Find the square root of both sides.

$$[H^+]^2 = 10^{-2}$$

$$[H^+] = 10^{-1}$$

Only the positive root is used here. The concentration of hydrogen ion, $[H^+]$, cannot have a negative value.

(f) First subtract the 8 from both sides. Then find the cube root of both sides.

$$y^3 + 8 = 0$$
$$y^3 = -8$$
$$y = -2$$

9.10(a) Group all terms on one side in the proper form for the quadratic equation.

$$x^2 = 5x - 6$$
$$x^2 - 5x + 6 = 0$$

The coefficient of x^2 is 1, of x is -5, and the remaining term is 6, so $a = 1$, $b = -5$, $c = 6$.

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-5) \pm \sqrt{(-5)^2 - 4\,(1)(6)}}{2\,(1)} \\ &= \frac{+5 \pm \sqrt{25 - 24}}{2} \\ &= \frac{+5 \pm 1}{2} \end{aligned}$$

The two roots are

$$\frac{+5 + 1}{2} = \frac{6}{2} = 3$$

and

$$\frac{+5 - 1}{2} = \frac{4}{2} = 2$$

9.11(a) Let $x = [CN^-]$. Since $[H^+] = [CN^-]$, $[H^+] = x$.

$$\frac{x\,(x)}{1.00 \times 10^{-2}} = 4.0 \times 10^{-10}$$

$$x^2 = 4.0 \times 10^{-10}(1.00 \times 10^{-2}) = 4.0 \times 10^{-12}$$

$x = 2.0 \times 10^{-6}$ Take the square root of both sides.

$[CN^-] = 2.0 \times 10^{-6}$ Omit the negative root, since the concentration of cyanide cannot be negative.

CHAPTER 10

ADDITIONAL TOPICS

Not all of the mathematical techniques used in chemistry have been discussed in detail in this book. The sections that follow contain information that might prove useful.

10.1. MORE RULES OF MATHEMATICS

Some fundamental rules of mathematical operations are given in Section 2.2. Others have been given at various places in the text, as need arose. A number of widely used procedures are listed and illustrated in Table 10.1.

TABLE 10.1

Rule or Pattern	*Examples*
$ax + bx = (a + b)\,x$	$2x + 3x = (2 + 3)\,x = 5x$ or $2\,(10) + 3\,(10) = 5\,(10)$ $20 + 30 = 50$
$ax - bx = (a - b)\,x$ Same as above, but b is now negative. In the following examples, subtraction will not be shown explicitly; it follows the same rules as addition.	

TABLE 10.1 (continued)

Rule or Pattern	Examples
$a(x + y) = ax + ay$ and conversely	$7(x + y) = 7x + 7y$ or $5(10 + 2) = 5(10) + 5(2)$; $5(12) = 50 + 10$; $60 = 60$
$ax + ay = a(x + y)$ This is called *factoring*, since it involves removing the factor a from all terms.	$9x + 9y = 9(x + y)$
$(a + b)(x + y) = a(x + y) + b(x + y)$ A special case is $(a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2$	$(4 + 3)(2 + 7) = 4(2 + 7) + 3(2 + 7)$; $7(9) = 4(9) + 3(9)$; $63 = 36 + 27$; $63 = 63$
$\frac{ax}{a} = x$ (canceling)	$\frac{2a}{2} = a$ $\quad \frac{3(4)}{3} = 4$; $4 = 4$
$\frac{x + y}{a} = \frac{x}{a} + \frac{y}{a}$	$\frac{6 + 8}{2} = \frac{6}{2} + \frac{8}{2}$; $\frac{14}{2} = 3 + 4$; $7 = 7$
Only the numerator, *never the denominator*, may be so divided.	
$\frac{a}{x + y} \neq \frac{a}{x} + \frac{a}{y}$	$\frac{1}{2 + 8} \neq \frac{1}{2} + \frac{1}{8}$; $\frac{1}{10} \neq \frac{5}{8}$
$\sqrt{x}\sqrt{y} = \sqrt{xy}$ but $\sqrt{x} + \sqrt{y} \neq \sqrt{x + y}$	$\sqrt{4}\sqrt{9} = \sqrt{36}$; $2(3) = 6$; $\sqrt{4} + \sqrt{9} \neq \sqrt{13}$; $2 + 3 \neq 3.6$
$x^a \cdot x^b = x^{a+b}$	$10^2 \cdot 10^3 = 10^5$; $100 \cdot 1000 = 100{,}000$
$\frac{x^a}{x^b} = x^{a-b}$	$\frac{2^3}{2^2} = 2^{3-2} = 2^1 = 2$; $\frac{8}{4} = 2$
$(x^a)^b = x^{ab}$	$(10^3)^2 = 10^6$; $(1000)^2 = 1{,}000{,}000$

10.2. ADDITION AND SUBTRACTION OF FRACTIONS

One of the rules in Table 10.1 indicated that a fraction with two terms in the numerator can be separated into two fractions (see p. 41), but a fraction with two terms in the denominator cannot be separated. Furthermore, Section 2.5 did not discuss ways to add and subtract fractions.

If you must add or subtract fractions, you can do so only if the fractions have the same denominator, called a common denominator. What can be done if fractions do not have the same denominator and still must be added or subtracted? The dilemma can be resolved by converting the original fractions to others with the *same value* but having a common denominator. To keep the numbers as small and as simple as possible, it is usually advantageous to use the smallest common denominator available. (This is why you were taught in elementary school how to find the least common denominator for fractions.)

10.2.A. Adding and Subtracting Fractions with a Common Denominator

To add or subtract fractions once they have a common denominator, add or subtract the *numerators*, with no change in the denominator. If this sounds illogical to you, imagine some apples cut in half. Take one half-apple. Then take two more half-apples. You now have three half-apples, that is, three pieces each of which is a half, $1/2 + 2/2 = 3/2$. The size of the pieces, shown by the denominator, is not changed in the addition and subtraction. The process of converting to a common denominator essentially means getting all the pieces the same size before you begin.

10.2.B. Converting to a Common Denominator

The procedure for converting a fraction to another one with the same value is to multiply both the numerator and the denominator by the same quantity. This is, of course, equivalent to multiplying by 1, which does not change the value. Such multiplication necessarily changes not only the denominator, but also the numerator.

■ EXAMPLE 1

Convert the fractions 1/2 and 1/6 to fractions with a common denominator.

An inspection of both denominators shows that one, the 2, can be converted to the other, 6, by multiplying by 3. If the denominator is

multiplied by 3, the numerator must also be multiplied by 3 to avoid changing the value of the fraction.

$$\frac{1}{2} \times \frac{3}{3} = \frac{3}{6}$$

This fraction has the same denominator as 1/6. ■

EXAMPLE 2

Convert the fractions x/y and $1/xy$ to fractions with a common denominator. Here both fractions have the factor y in the denominator, but only one has the factor x. Therefore, to make the denominators the same, the other must also have the factor x. Since the numerator and denominator must be multiplied by the same thing, the fraction x/y must be multiplied by x/x. Then

$$\frac{x}{y}\left(\frac{x}{x}\right) = \frac{x^2}{xy}$$

with the same denominator as $1/xy$. ■

EXAMPLE 3

Convert the fractions 1/3 and 1/4 to fractions with a common denominator.

Here there is no factor common to both denominators. Therefore, both fractions will have to be converted to new ones with a denominator in common. The smallest number containing the factors 3 and 4 (the denominators already present in the two fractions) is 12. Hence 12 must be the new denominator. To convert 3 to 12, it is necessary to multiply by 4. To convert 4 to 12, it is necessary to multiply by 3. For each fraction the numerator must be multiplied by the same quantity as the denominator.

$$\frac{1}{3}\left(\frac{4}{4}\right) = \frac{4}{12}$$

$$\frac{1}{3}\left(\frac{3}{3}\right) = \frac{3}{12}$$ ■

EXAMPLE 4

Convert fractions $1/x$ and $1/y$ to fractions with a common denominator.

Here again, there is no common factor in the denominators, and the lowest quantity containing both the factors x and y is xy. Hence the

fraction $1/x$ must be multiplied by y/y and $1/y$ must be multiplied by x/x.

$$\frac{1}{x}\left(\frac{y}{y}\right) = \frac{y}{xy}$$

$$\frac{1}{y}\left(\frac{x}{x}\right) = \frac{x}{xy}$$

Note that this is just like Example 3 except that letters are used instead of numbers. ■

■ EXAMPLE 5

Convert 10 and x/y to fractions with a common denominator.

Only the denominator of the fraction needs to be considered in finding the common denominator, since a whole number can be considered as a fraction with a denominator of 1.

Multiply the whole number 10 by y/y, which has the same denominator as x/y.

$$10\left(\frac{y}{y}\right) = \frac{10y}{y}$$ ■

■ EXAMPLE 6

Add or subtract as indicated.

(a) $\dfrac{3}{4} + \dfrac{1}{4}$

(b) $\dfrac{1}{2} - \dfrac{1}{6}$

(c) $\dfrac{x}{y} + \dfrac{1}{xy}$

(d) $\dfrac{1}{3} + \dfrac{1}{4}$

(e) $\dfrac{1}{x} - \dfrac{1}{y}$

(f) $1 + \dfrac{x}{y}$

Note that the first pair of fractions already has a common denominator. The other numbers are exactly the ones used in Examples 1, 2, 3, 4,

and 5, so we have already found the common denominators; the answers from the previous examples will be used here.

(a) $\frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$. It is useful to simplify the answer.

(b) $\frac{1}{2} - \frac{1}{6} = \frac{3}{6} - \frac{1}{6} = \frac{2}{6}$

(c) $\frac{x}{y} + \frac{1}{xy} = \frac{x^2}{xy} + \frac{1}{xy} = \frac{x^2 + 1}{xy}$

(d) $\frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12} = \frac{7}{12}$

(e) $\frac{1}{x} - \frac{1}{y} = \frac{y}{xy} - \frac{x}{xy} = \frac{y - x}{xy}$

(f) $1 + \frac{x}{y} = \frac{y}{y} + \frac{x}{y} = \frac{y + x}{y}$ ■

PROBLEMS

10.1 Convert to fractions with a common denominator.

(a) $\frac{1}{5}$ and $\frac{3}{10}$ *(b) $\frac{4}{x}$ and $\frac{a}{2}$

(c) $\frac{a}{x}$ and $\frac{b}{y}$ *(d) 5 and $\frac{2}{3}$

(e) $\frac{1}{A}$ and $\frac{1}{B}$ (f) A and $\frac{1}{B}$

10.2 Add or subtract as indicated (see Problem 10.1).

(a) $\frac{1}{5} + \frac{3}{10}$ *(b) $\frac{4}{x} + \frac{a}{2}$ (c) $\frac{a}{x} - \frac{b}{y}$

(d) $5 - \frac{2}{3}$ (e) $\frac{1}{A} - \frac{1}{B}$ (f) $A + \frac{1}{B}$

10.3. INTRODUCTION TO TRIGONOMETRY

Trigonometry is a branch of mathematics that studies triangles. This may sound as if it is of limited use, but in fact the principles of trigonometry are used extensively. The numerical values of some trigonometric

functions appear in many kinds of equations. You can use trigonometry to determine a distance indirectly, by measuring a distance that is at an angle to the distance you must calculate.

This section will not provide any comprehensive coverage of trigonometry. It will introduce two important functions and show how these can be used for calculating distances.

In a triangle with a right angle (Figure 10.1), the side opposite the right angle, c, is called the *hypotenuse*. For the other angles, such as θ (theta), the sine is defined as the ratio of the length of the side opposite that angle to the length of the hypotenuse.

$$\sin\theta = \frac{a}{c} = \frac{\text{side opposite}}{\text{hypotenuse}}$$

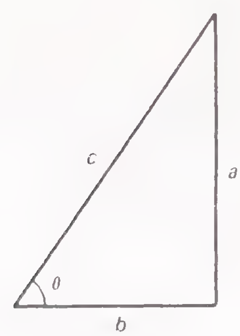


FIGURE 10.1

The cosine is defined as the ratio of the length of the adjacent side to that of the hypotenuse.

$$\cos\theta = \frac{b}{c} = \frac{\text{side adjacent}}{\text{hypotenuse}}$$

These ratios depend only on the size of the angles, not on the actual length of the sides. That is, for a given angle θ, if the length of the hypotenuse is doubled, the lengths of the other two sides also double, so the ratio is the same. Tables giving the numerical values of these ratios are available in handbooks. They are also included on some calculators.

The reason these trigonometric ratios are useful is that it is often necessary to measure a distance indirectly. If you can draw a right triangle in which one of the sides is the needed length and another is a length you can measure, and if you can also measure one of the angles, you can calculate the needed length by using the appropriate trigonometric function.

EXAMPLE 7

Find the length d in Figure 10.2 if the length a is 2.00 cm and the angle θ s 43°.

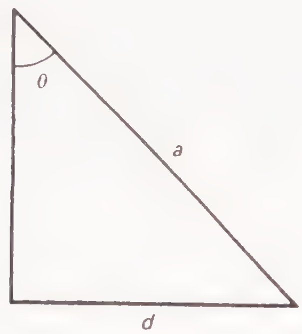


FIGURE 10.2

Since d is the side opposite the angle θ and a is the hypotenuse of the right triangle, the function relating them is the sine.

$$\frac{d}{a} = \sin\theta$$

$$\frac{d}{2.00\ \text{cm}} = \sin 43^\circ$$

From a table of trigonometric functions, $\sin 43.0^\circ = 0.682$.

$$\begin{aligned} d &= 2.00\ \text{cm}\ (\sin 43^\circ) = 2.00\ \text{cm}\ (0.682) \\ &= 1.36\ \text{cm} \end{aligned}$$

The smaller an angle θ, the smaller the side opposite and the longer the side adjacent in proportion to the hypotenuse. (Draw a few right triangles and see.) Therefore, for small angles the sine is smaller (near 0) and the cosine is large (near 1). No side can be larger than the hypotenuse, so the ratio cannot be more than 1. For angles approaching 90°, the sine is large and the cosine small. Therefore, if you have the equation

$$r = d\sin\theta$$

r is small if θ is small, but r is nearly equal to d if θ is near 90°, where $\sin\theta$ is near 1. ■

SOLUTIONS TO STARRED PROBLEMS

10.1(b) There are no common factors in the denominators of $4/x$ and $a/2$, so both fractions have to be converted to new ones to achieve a common denominator of $2x$.

$$\frac{4}{x}\left(\frac{2}{2}\right) = \frac{8}{2x} \quad \text{and} \quad \frac{a}{2}\left(\frac{x}{x}\right) = \frac{ax}{2x}$$

(b) The whole number 5 has, in effect, a denominator of 1. To convert it to a denominator of 3, multiply by 3/3.

$$\frac{5}{1} \times \frac{3}{3} = \frac{15}{3}$$

10.2(b) The fractions were converted to a common denominator in Problem 10.1(b). Using those fractions, add the numerators.

$$\frac{4}{x} + \frac{a}{2} = \frac{8}{2x} + \frac{ax}{2x} = \frac{8 + ax}{2x}$$

ANSWERS TO PROBLEMS

CHAPTER 1

1.1	See Table 1.2.		
1.2	(a) 25 cm	(b) 250 mm	(c) 0.372 g
	(d) 293 mL	(e) 0.029 kg	(f) 2000 g
	(g) 2.932 km		
1.3	(a) 0.525 L	(b) 525 cm^3	
	(c) 1320 mL	(d) 1320 cm^3	
1.4	(a) 24 g	(b) 6 g	
	(c) No. 15 g is close to 1 mole.		
	(d) Answer: (3)		
1.5	(a) 2	(b) 5	(c) 1
	(d) 3	(e) 4	
1.6	(a) 152 g	(b) 0.150 g	(c) 12 mL
1.7	(a) 4400 or 4.4×10^3	(b) 3500 or 3.5×10^3	
	(c) 2.00	(d) 300 or 3×10^2	

CHAPTER 2

2.1	(a) $+7$	(b) -1	(c) 0
	(d) -26	(e) $+4x$	(f) $-x$
2.2	(a) 5	(b) 11	(c) -5
	(d) 11	(e) 11	(f) 11
	(g) $5x$	(h) $3a$	
2.3	(a) $+16$	(b) -16	(c) -20
	(d) $+1$	(e) -2	(f) -60

(g) $-2x$ (h) $+20x$ (i) $-\frac{3}{2}$ or -1.5
(j) $+3$ (k) $-3x$ (l) x
(a) -1 (b) -2 (c) $+2$
(d) 0 (e) -1 (f) $+1$
(g) -3 (h) $+3$
(a) One Ca^{2+} with two Cl^-; $CaCl_2$
(b) Two Na^+ with one S^{2-}; Na_2S
(c) One Al^{3+} with three I^-; AlI_3
(d) One Mg^{2+} with one O^{2-}; MgO
(a) Two Cr^{3+} with three O^{2-}; Cr_2O_3
(b) Two As^{3+} with three S^{2-}; As_2S_3
(c) Three Mg^{2+} with two N^{3-}; Mg_3N_2
(d) Two P^{5+} with five O^{2-}; P_2O_5
(e) Three O^{2-} with two Al^{3+}; Al_2O_3
(a) $+300$ J (b) -243 J
(a) -2 (b) -79 (c) $+20$
(d) $-3x$ (e) -14
(f) $-(x + y) = -x - y$
(g) $-(x - 2) = -x + 2$
(h) $-(x - 7y) = -x + 7y$
(i) $-x^2$

(a) $\frac{1}{m}$ (b) $\frac{1}{4}$ (c) $\frac{1}{25}$

(d) $\frac{1}{3x^2}$ (e) $\frac{1}{100} = 0.01$

(f) $\frac{1}{0.01} = 100$ (g) $\frac{1}{0.5} = 2$

(a) 4 (b) $\frac{8}{2} = 4$ (c) $\frac{7}{9}$

(d) x (e) x^2 (f) $\frac{2y}{3x}$

(a) Multiply by 2 and divide by 3.
(b) Multiply by a and divide by b.
(c) Divide by 10.
(d) Multiply by 10 and divide by 2.
(e) Divide by x.
(f) Multiply by x and divide by 5.

(a) $\frac{8}{27}$ (b) $\frac{x}{6}$

(c) $\frac{18}{6} = 3$ (d) $\frac{6}{x}$

(e) $\frac{2\,(a + b)}{y}$ or $\frac{2a + 2b}{y}$

(f) $\dfrac{1}{2\,(a+b)}$ or $\dfrac{1}{2a+2b}$

(g) 1 (h) $\dfrac{1}{x^2}$

(i) 7.2 g (j) 25 cm^3

2.13 (a) $\dfrac{1}{4}$ (b) $\dfrac{2}{20} = \dfrac{1}{10}$ (c) $\dfrac{3}{50}$

(d) $\dfrac{x}{ay}$ (e) $\dfrac{x}{xy} = \dfrac{1}{y}$ (f) $\dfrac{x}{y^2}$

(g) $\dfrac{2x}{2y^2} = \dfrac{x}{y^2}$

(h) $\dfrac{a}{b\,(x+y)}$

(i) $\dfrac{1}{a\,(a+b)}$

2.14 (a) 4 (b) 20 (c) 4 (d) $\dfrac{ac}{b}$

(e) $\dfrac{15}{2}$ (f) g (g) mole

2.15 (a) 1 (b) 5

(c) $\dfrac{1}{2}$ (d) $\dfrac{x^2}{y^2}$

(e) $\dfrac{x\,(a+b)}{y}$

(f) $\dfrac{x+y}{y}$ (g) $\dfrac{g}{L}$

2.16 $\dfrac{[Ag^+][HNO_2]}{[H^+]}$

2.17 $\dfrac{[A^-]}{[HA][OH^-]}$

2.18 (a) $\dfrac{1}{2}$ (b) 5 (c) 4

(d) $\dfrac{36}{5}$ (e) Cannot be simplified.

(f) $\dfrac{b}{c}$ (g) $\dfrac{a}{c}$ (h) $\dfrac{ab}{c}$

(i) $\dfrac{xy}{z}$ (j) $x+y$ (k) $\dfrac{1}{x+y}$

(l) Cannot be simplified.

(m) $a+b$

2.19 (a) 2 (b) 3 ft (c) 7 cm^2
(d) 1.0 m^2 (e) 15 cm

2.20 (a) $\frac{x}{x} + \frac{y}{x} = 1 + \frac{y}{x}$ (b) $\frac{9}{3} + \frac{x}{3} = 3 + \frac{x}{3}$

(c) $\frac{9}{x} - \frac{x}{x} = \frac{9}{x} - 1$ (d) $\frac{9}{x} + \frac{x^2}{x} = \frac{9}{x} + x$

(e) $\frac{\pi r^2}{r^3} + \frac{r^2 h}{r^3} = \frac{\pi}{r} + \frac{h}{r}$

2.21 (a) Less than 1 (b) 1
(c) More than 1 (d) 1
(e) More than 1 (f) More than 1
(g) Less than 1 (h) 1
(i) Less than 1 (j) More than 1
(k) 1 (l) 1

2.22 (a) $\frac{\text{20 children}}{\text{8 families}}$ = 2.5 children per family

(b) $\frac{\text{400 students}}{\text{22 faculty}}$ = 18.1 students to every faculty member

(c) $\frac{\text{50 boys}}{\text{40 girls}}$ = 1.25 boys to every girl

(d) $\frac{\text{20 nails}}{\text{5 shelves}}$ = 4 nails for every shelf

2.23 (a) $\frac{20\ \text{H}}{10\ \text{O}}$ = 2 H to 1 O (H_2O)

(b) 20 C : 60 H : 10 O = 2 C : 6 H : 1 O (C_2H_6O)
(c) 50 Na : 20 Cr : 350 O = 2 Na : 2 Cr : 7 O ($Na_2Cr_2O_7$)

(d) $\frac{16\ \text{Fe}}{24\ \text{O}} = \frac{2\ \text{Fe}}{3\ \text{O}}$ (Fe_2O_3)

2.24 (a) cost = \$1.25 (number of cans)
(b) heat = 4.18 J (degrees)
(c) heat = 44.7 KJ (grams burned)

2.25 60%

2.26 (a) 77.7% (b) 63.6%
(c) 70.0% (d) 37.0%

2.27 35.7%

2.28 (a) 46.0 g (b) 92.0 g (c) 69.0 g
(d) 9.20 g (e) 0.272 g

2.29 (a) 5.89 g (b) 105 g (c) 0.126 g

2.30 4.1%

2.31 1.6%

2.32 The percent average deviation was 0.25%, well below 1%.

CHAPTER 3

3.1 (a) 2×10^3 (b) 2×10^{-3}
(c) 3.45×10^5 (d) 3.45×10^{-4}
(e) 5×10^9 (f) 7.5×10^{-6}
(g) 2.7×10^1 (h) 9.32×10^2
(i) 2.973×10^{-3}

3.2 (a) 0.00003 (b) 0.0072 (c) 0.0000005
(d) 9,100,000 (e) 8.2 (f) 298
(g) 0.0298 (h) 37.9

3.3 (a) 10^4 (b) 1 (c) 10^{12}
(d) 10^{-2} (e) 10^2 (f) 10^{-12}
(g) 10^{-1} (h) 10^{127} (i) x^8
(j) x^{m+n}

3.4 (a) 10^{-1} (b) 10^1 (c) 10^{-18}
(d) 10^7 (e) 10^{23} (f) 10^{25}
(g) 10^{-25} (h) 10^{120} (i) e^{H-T}
(j) x^{m-n}

3.5 (a) 8×10^9 (b) 8×10^{-3}
(c) $32 \times 10^{12} = 3.2 \times 10^{13}$
(d) $12 \times 10^1 = 1.2 \times 10^2$
(e) $74 \times 10^{-8} = 7.4 \times 10^{-7}$
(f) 9.0×10^{23}
(g) $45 \times 10^{-3} = 4.5 \times 10^{-2}$
(h) $24 \times 10^{-5} = 2.4 \times 10^{-4}$
(i) $38 \times 10^{-10} = 3.8 \times 10^{-9}$

3.6 (a) 2×10^3
(b) $0.5 \times 10^9 = 5 \times 10^8$
(c) $0.75 \times 10^{-12} = 7.5 \times 10^{-13}$
(d) 2×10^9
(e) 3.0×10^{17} (f) 3×10^9
(g) 5.21×10^{-23} (h) 4.0×10^{-6}
(i) 2×10^5 (j) 5×10^{-16}

3.7 (a) 7.5×10^{-5} (b) 4×10^9
(c) $0.5 \times 10^5 = 5 \times 10^4$
(d) 7×10^5
(e) 4×10^{-3}
(f) 2.45×10^2

3.8 (a) 3.34×10^2 (b) 3.2×10^3 (c) 1.8×10^{16}
(d) 2.7×10^9 (e) 2.1×10^{-11} (f) 1.1

3.9 (a) 7.42×10^{-3} mole (b) 3.0×10^{-3} mole

3.10 (a) 1.59×10^{-10} (b) 1.6×10^{-10} [compare with (a)]
(c) 1.2×10^{-23} (d) 3.6×10^{-4}
(e) 3.5×10^{-4} (f) 6×10^{-8}

3.11 (a) 10^{12} (b) 10^{-12} (c) 10^{12}
(d) 10^{12} (e) 10^2 (f) e^4
(g) x^{15} (h) x^{-15}

3.12 (a) 9×10^{18} (b) 9×10^{-18} (c) 3.43×10^8
(d) 3.2×10^{-19} (e) 8.1×10^{15}

3.13 (a) $x = 4.0 \times 10^{10}$, $y = 2.0 \times 10^{8}$
(b) $x = 2.0 \times 10^{5}$, $y = 2.5 \times 10^{-1}$
(c) $x = 4.0 \times 10^{-7}$, $y = 2.0 \times 10^{-17}$
(d) $x = 5.0 \times 10^{-12}$, $y = 1.7 \times 10^{-4}$

3.14 (a) 1.4×10^{-8} (b) 1.4×10^{-8} [same as (a)]
(c) 3.4×10^{-11} (d) 1.7×10^{-49}
(e) 3.7×10^{-15}

3.15 (a) $\pm 10^{3}$ (b) 10^{-2} (c) 10^{3}
(d) $\pm 3.2 \times 10^{4}$ (e) 2.15×10^{4}
(f) 7.96 (g) $\pm 2.0 \times 10^{-1}$
(h) $\pm 4.0 \times 10^{-1}$ (i) $\pm 2.7 \times 10^{-2}$
(j) 3.0×10^{-4}

3.16 (a) 5.40×10^{3} (b) 6.83×10^{6} (c) 1.88×10^{-4}
(d) 1.099×10^{2} (e) 7.05×10^{-1} (f) 4.95×10^{4}
(g) 3.4×10^{-2}

CHAPTER 4

4.1 (a) No. The reported temperature is higher than the boiling point of water, so water could not be heated that hot. Also, the reported temperature is higher than the temperature of the hot water, but the temperature of the mixture must be between the temperature of the hot and the cold water.
(b) No. The temperature of the mixture must be between the temperature of the hot and the cold water, not colder than both of them.

4.2 (a) No. The weight of the compound can be no more than the sum of the weights of the components.
(b) Yes. Some of the iron or the sulfur might not have reacted, so the weight of the compound can be smaller than the sum of the weights of iron and sulfur.
(c) The answer is possible, as in part (b), but is unlikely from the description of the experiment. The compound of magnesium and oxygen would be expected to weigh more than the magnesium used to form it.

4.3 (a) Yes. The units for the answer will be g, an appropriate unit of mass.
(b) No. The units for the answer will be g^2/cm^3, not a unit of volume.
(c) No. 20.7 oz is a little over 1 lb, not 331 lb.
(d) No. There are 25.00 g of water in 25.00 mL of water. It is not likely that 765 g of acetic acid would fit into only 25.00 mL.

4.4 (a) $4 \times \sim\$10 \approx \$40$ (b) $8 \times \sim\$1 \approx \$8$
(c) If you round down to \$2, the total would be about \$10. If you round up to \$3, the total would be about \$15. Therefore, the total is between \$10 and \$15. Alternatively, think of the problem as five items at \$2 and five at \$0.50. Total \$13.

4.5 (a) 4 @ 50 mL = 200 mL (Use the larger volume.) 3 @ 40 mL = 120 mL. A minimum of 320 mL is needed. You should mix 350–400 mL to have enough for an extra sample if needed.
(b) About 6 g.

4.6 (a) No. Since 273 is less than 298, multiplication by 273/298 should give a result smaller than the original number.
(b) Yes. The fraction is greater than 1, so the answer should be greater than the original number.
(c) No. See part (b).
(d) No. The answer looks much too big. An approximate mental calculation shows that the answer should be somewhat over 10. (The answer shown is the result of multiplying by 500 instead of dividing.)
(e) No, the decimal point is in the wrong place. See part (d).

4.7 (a) Answer (2) (b) Answer (2)

4.8 (The results of actual calculations are given. You cannot expect to get the identical answer from an approximate calculation, but your answers should be within 10% of the ones shown.)
(a) 330 (b) 508 (c) 22.9 (d) 630 (e) 1.03

4.9 (a) 700 − 8 (b) 300 + 9
(c) \$3 − 0.05
(d) \$11 − 0.02 or \$10 + \$1 − 0.02
(e) 90 − 1 or 100 − 11
(f) 80 − 3 or 70 + 7
(g) 20 + 3
(h) 40 + 6 or 50 − 4
(i) 20 − 2
(j) \$2 − 0.11

4.10 (a) 267 (b) \$9.45 (c) \$13.86
(d) \$31.60 (e) 2160 (f) 2640

4.11 (a) 1800 (b) 8.8 (c) 700
(d) 4.8 (e) 30 (f) 2300

CHAPTER 5

5.1 (a) 0.029 kg (b) 0.208 L (c) 153 g
(d) 150 cm (e) 9080 g (f) 3.2 qt
(g) 2.8 L (h) 1.6 cm

5.2 (a) 153,000 mg or 1.53×10^5 mg (b) 9.08 kg
(c) 183 cm (d) 0.33 g

5.3 (a) 236 mL (b) 15 mL (c) 5.0 mL
(d) 30 mL

5.4 (a) 91.4 m (b) 402 m (c) 1.61×10^3 m

5.5 (a) 109 yd (b) 1.09×10^3 yd (c) 5.47×10^3 yd

5.6 (a) 3.73 mi (b) 6.21 mi

5.7 (a) 0.30 mi (b) 490 m

5.8 (a) 68.3 mi/hr (b) 89 km/hr (c) 51 ft/sec

5.9 (a) 1.86×10^5 mi/sec (b) 6.70×10^8 mi/hr
(c) 1.29 sec

5.10 (a) 40 g (b) 2.0 ml (c) 30 ml

5.11 (a) 1025 g (b) 967 g (c) 976 ml (d) 1000 ml

5.12 (a) 2.0 moles (b) 1.0 mole (c) 0.65 mole
(d) 2.5×10^{-4} mole (e) 7.0×10^{-3} mole
(f) 1.33 moles (g) 17.9 moles

5.13 (a) 0.1 mole or 10^{-1} mole (b) 2.0 moles (c) 0.5 mole

5.14 (a) 96 g (b) 560 g (c) 0.044 g
(d) 22 g (e) 160 g (f) 0.48 g

5.15 (a) 6 moles (b) 18 moles (c) 0.5 mole
(d) 0.1 mole (e) 1.2 moles (f) 10^{-4} mole

5.16 (a) 0.17 L, 170 mL (b) 2 L, 2000 mL
(c) 10^{-2} L, 10 mL (d) 0.33 L, 330 mL
(e) 0.33 L, 330 mL (f) 0.11 L, 110 mL
(g) 1.1×10^{-4} L, 0.11 mL, or 1.1×10^{-1} mL

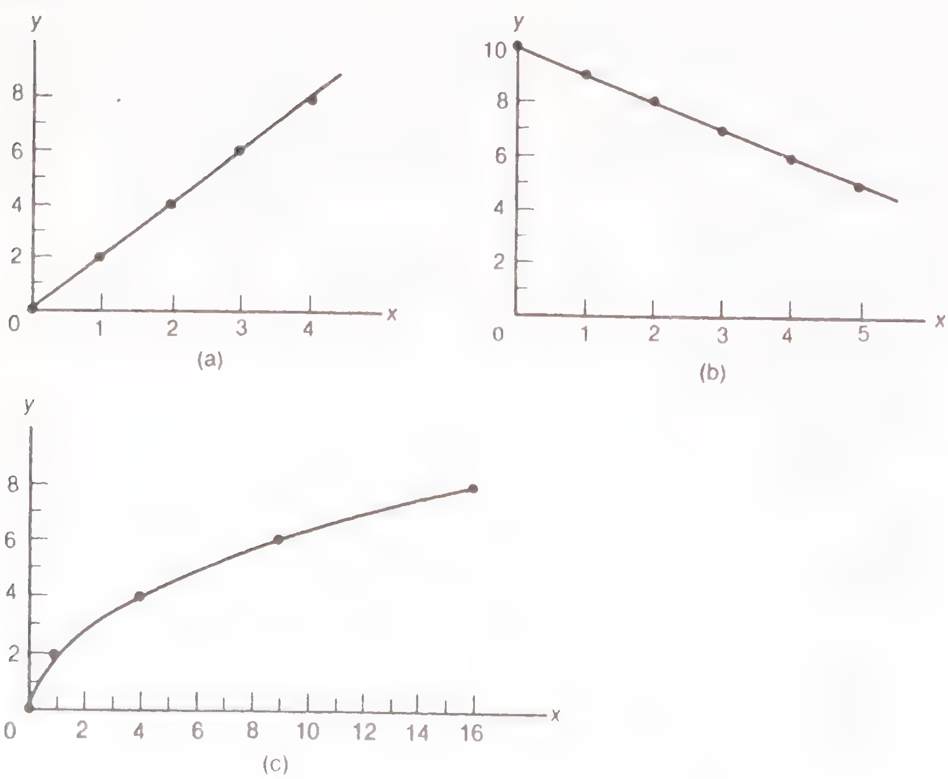
5.17 (a) 5.0 g (b) 1.0 g (c) 0.20 g
(d) 150 g (e) 500 g (f) 2.9×10^3 g

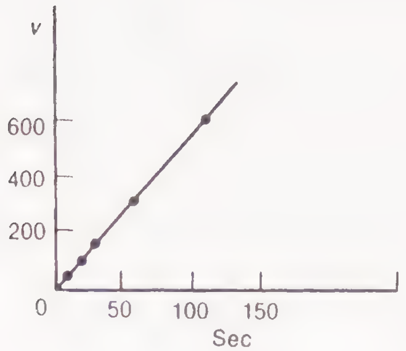
5.18 (a) 26 moles of alcohol, 26 moles of CO_2
(b) 0.84 mole, 39 g (c) 0.84 mole, 37 g
(d) 580 L, 19 L

5.19 (a) 0.50 mole of $K_2Cr_2O_7$, 2.0 moles of H_2SO_4
(b) 87 g (c) 17 g
(d) 0.40 mole; 22 mL (e) 49 g

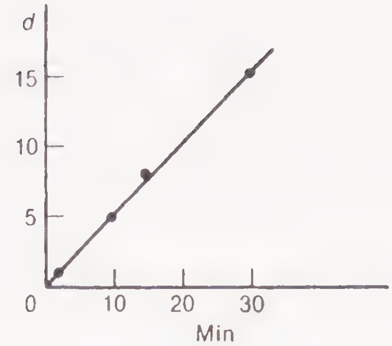
CHAPTER 6

6.1



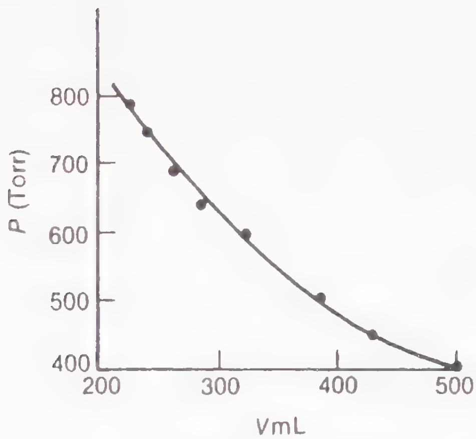


(d)

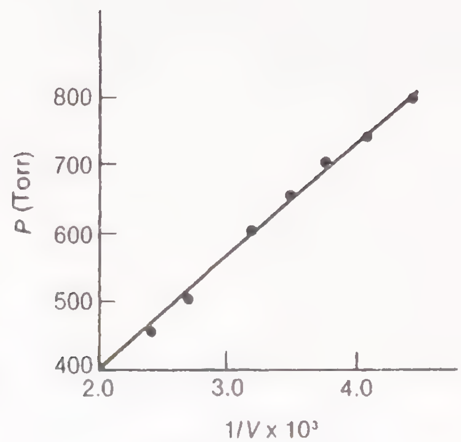


(e)

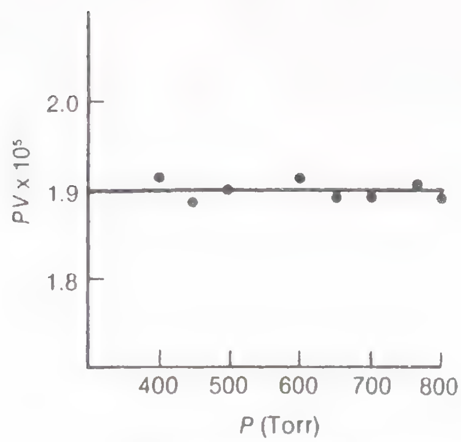
6.2



(a) Graph of P versus V



(b) Graph of P versus $1/V$



(c) Graph of PV versus P

Comments:

(a) V increases as P decreases, but not in a straight line.

(b) $1/V$ decreases in a straight line as P decreases. $P \propto 1/V$.

(c) The product PV is constant; it does not change as P changes.

6.3 (a) As x increases, y increases.
(b) As x increases, y decreases.
(c) As x increases, y increases up to a point, then levels out and does not change.
(d) As the time increases, y increases for a while, then levels out for a while. Then, at a later time, y starts to increase again.
(e) As x increases, y increases, slowly at first, then at larger values of x very rapidly.
(f) As x increases, y increases very rapidly at first, then more slowly.
(g) As x increases, y decreases, very rapidly at first, then more slowly, and finally very slowly.
(h) There is no consistent pattern.
(i) At short distances, a small increase in distance results in a large decrease in energy. At a given distance, the drop in energy stops, and at longer distances the energy is higher. This levels out, and at some distance the energy stops changing and is the same at all longer distances.

6.4 (a) Slope = 2. Equation: $y = 2x$.
(b) Slope = -1. Equation: $y = -x + 10$ or $x + y = 10$.
(c) Not a straight line.
(d) Slope = 5. Equation: $v = 5t$.
(e) Slope = 2. Equation: $d = 2t$.

CHAPTER 7

7.1 (a) 4 (b) 10 (c) $27 - y$
(d) 16 (e) $3y$ (f) 13
(g) 7 (h) 3 (i) 1.6

7.2 (a) $\dfrac{27 - y}{3}$ or $9 - \dfrac{y}{3}$ (b) $\dfrac{15 + 2y}{5}$ or $3 + \dfrac{2y}{5}$
(c) $2 - 2y$ (d) $0.5 - yz$
(e) $\dfrac{b - yz}{a}$ (f) $\dfrac{yz + 3}{a}$
(g) $\dfrac{bz + y}{a}$ (h) $\dfrac{bz}{ay}$
(i) $\dfrac{3ay}{2z}$

7.3 (a) $27 - 3x$ (b) $\dfrac{5x - 15}{2}$ (c) $\dfrac{6 - 3x}{6}$ or $1 - \dfrac{x}{2}$
(d) $\dfrac{0.5 - x}{z}$ (e) $\dfrac{b - ax}{z}$
(f) $\dfrac{ax - 3}{z}$ (g) $ax - bz$
(h) $\dfrac{bz}{ax}$ (i) $\dfrac{2xz}{3a}$

7.4 (a) $[OH^-] = 10^{-7}$ (b) $[H^+] = 10^{-1}$
(c) $[Mg^{2+}] = 3.3 \times 10^{-7}$

7.5 (a) 13 (b) 16 (c) 4
(d) $30 + 2y$ (e) $\frac{a}{b}$ (f) $\frac{y}{a}$
(g) $\frac{1}{7ab^2}$ (h) $\frac{byz}{a}$ (i) 108
(j) -40 (k) 25 (l) 77
(m) -40

7.6 (a) $\frac{90 - 2y}{3}$ or $30 - \frac{2y}{3}$ (b) -4
(c) $\frac{4}{8}$ or 0.5 (d) $-y$
(e) $-\frac{yT}{T - y}$ or $+\frac{yT}{y - T}$
(f) $cz - \frac{az}{b}$ or $\frac{bcz - az}{b}$
(g) $0.32 - 0.59\,n$ (h) $1.8 - 0.22\,f$
(i) $\frac{1}{y - 3}$ (j) $\frac{y}{y - 1}$

7.7 (a) 0.5 L (b) 6.4×10^{-3} mol (c) 1.0 mol
(d) 8.9 g (e) 25 cm^3 (f) 1.5 atm
(g) 10 L

7.8 (a) $a = 3.3$ (b) $c = 98$ (c) $d = \frac{4}{9} = 0.4$
(d) $[H^+] = 3.5 \times 10^{-4}$. If $[F^1] = [HF]$, they cancel and $[H^+]$ is 3.5×10^{-4}.
(e) $[HF] = 1.0$ (f) $[HF] = 2.6 \times 10^{-2}$
(g) $[H^+] = 5 \times 10^{-6}$

7.9 For (a) through (e), let x equal one number and y equal the other.
(a) $x + y = 23$ (b) $x - y = 6$ (c) $xy = 70$
(d) $x = 3y$ (e) $x = y + 5$

7.10 (a) Let s, b, and j equal the money contributed by Smith, Brown, and Jones, respectively, and t equal the total. Then $t = s + b + j$.
(b) Let t and h equal the ft of fence painted by Tom and Huck, respectively, and let f equal the total ft of fence painted. Then $f = t + h$.
(c) Let m = lb of meat. Let g = number of guests. Then $m = 0.5g$.
(d) Let b = bottles of wine. Let g = number of guests. Then $b = 1/3g$ or $3b = g$.
(e) Let r = number of rose bushes. Let b = number of flower beds. Then $r = 3b$.
(f) Let m = mass of iron. Let r = mass of ore. Then $m = 1/20r$.

(g) Let h = moles of hydrogen. Let z = moles of zinc. Then $h = z$.
(h) Let h = moles of hydrogen. Let x = moles of oxygen. Then $h = 2x$.
(i) Let c = number of chloride ions. Let z = number of zinc chloride units. Then $c = 2z$.
(j) Let p = number of phosphoric acid molecules left. Then $p = x - y$.

7.11 (a) P decreases. (b) V increases.
(c) P decreases. (d) I decreases.
(e) M decreases. (f) Mol increase.

7.12 (a) E increases. (b) P increases.
(c) Both increase: E by a factor of 2, P by a factor of 4. $(2I)^2 = 4I^2$.

7.13 (a) x decreases. They are not proportional.
(b) x decreases.

7.14 (a) Cannot be predicted from the equation.
(b) Negative.

7.15 (a) Decreasing the radius r will cause a proportional increase in the frequency v.
(b) Increasing the length l will cause a proportional decrease in the frequency v.
(c) Increasing the tension T will cause an increase in the frequency v proportional to the square root of the increase in T.

7.16 (a) D is directly proportional to C_{aq} and inversely proportional to C_{org}.
(b) I is directly proportional to E and inversely proportional to R.
(c) H is proportional to the entire quantity $(E + PV)$ but is not proportional to any one term.
(d) W is directly proportional both to P and to t.
(e) a is directly proportional to the term $(V_t - V_0)$ and inversely proportional to t. However, a is not proportional to either V_t or V_0 alone, unless the other one is 0.
(f) In any equation, π is a constant and not a variable. T is directly proportional to the square root of l and inversely proportional to the square root of g.
(g) Q is directly proportional to N and to m and directly proportional to the square of v.
(h) MR is directly proportional to M and to the term $(n^2 - 1)$. MR is inversely proportional to d and to the term $(n^2 + 2)$.

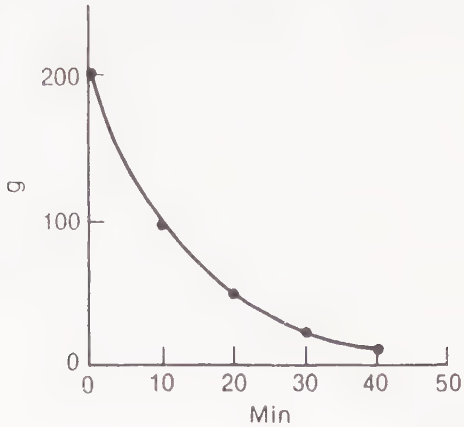
CHAPTER 8

8.1	(a) -2	(b) 9	(c) 130	(d) -3
8.2	(a) 9.4771	(b) 3.9542	(c) 3.7574	(d) 200.6684
	(e) 2.6684	(f) 3.7574	(g) 2.0969	(h) 0.3838
8.3	(a) -8.5229	(b) -1.1720	(c) -4.5513	(d) -2.0353
	(e) -12.2604	(f) -2.4935	(g) -0.8153	

8.4 (a)(b) No. If a number is between 1 and 10, its log is between 0 and 1.
(c) No. The log of a number less than 1 is negative.
(d) Yes. This could be correct.

8.5 (a) 1.16 (b) 0.89

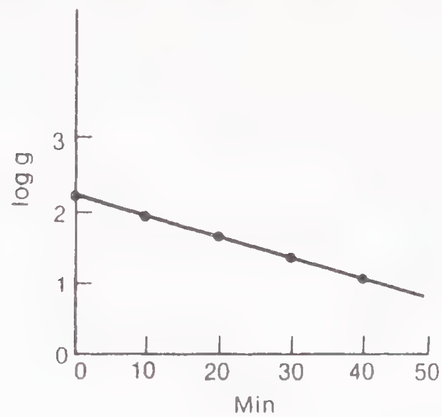
8.6 (a)



(a)

(b) mass	200	100	50	25	12.6 g
log g	2.30	2.00	1.70	1.40	1.10

(c)



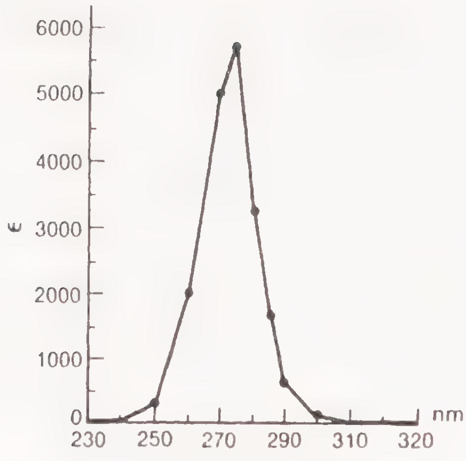
(b)

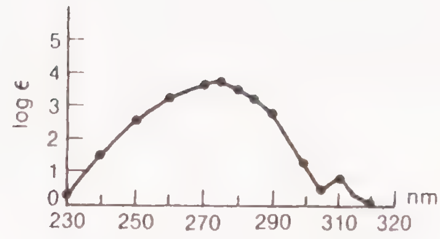
(d) slope $= -3.0 \times 10^{-2}$

8.7

ε	log ε
1.59	0.20
31.6	1.50
400	2.60
2000	3.30
5000	3.70
5620	3.75
3160	3.50

ε	log ε
1600	3.20
640	2.81
16.0	1.20
3.16	0.50
7.94	0.90
2.00	0.30
1.26	0.10





Plotting ε makes it hard to see the second peak, since the vertical axis must be compressed to accommodate the maximum value. Plotting log ε makes the peaks appear nearer the same size; this is easier to plot, but not the true relationship.

8.8 (a) 10^{10} (b) 10^2 (c) 10^{-1}
(d) 10^{-8} (e) 6.36×10^6 (f) 2.44
(g) 6.62×10^2 (h) 4.717×10^5
(i) 9.008

8.9 (a) 5.07×10^{-7} (b) 1.09×10^{-4}
(c) 1.55×10^{-1} (d) 5.61×10^{-3}
(e) 2.16×10^{-13}

8.10 (a) 3 (b) 0 (c) 8 (d) 4.70
(e) 2.15 (f) 0.52 (g) 2.70

8.11 (a) 10^{-5} (b) 10^{-11} (c) 1
(d) 6.3×10^{-4} (e) 3.6×10^{-10}
(f) 4×10^{-7} (g) 3.2×10^{-1}

8.12 (a) No (b) No (c) 3.38
(d) No (e) 1.18×10^{-9}

8.13 (a) 2.7×10^3 (b) $2.4 \times 10^1 = 24$
(c) 4×10^3 (d) $3.2 \times 10^1 = 32$
(e) 2.7×10^{-5} (f) 1.2×10^{-5}

8.14 (a) $1.7 \times 10^1 = 17$ (b) 1.7
(c) ± 3.1 (d) $\pm 1.1 \times 10^1 = \pm 11$
(e) $\pm 1.3 \times 10^2$ (f) ± 5
(g) ± 2.6

CHAPTER 9

9.1 (a) $F = hdgA$ (b) $d = \dfrac{P}{hg}$

9.2 (a) $\dfrac{P_1}{P_2} = \dfrac{n_1 T_1 V_2}{n_2 T_2 V_1} = \dfrac{M_2 V_2 T_1 w_1}{M_1 V_1 T_2 w_2}$

(b) $\dfrac{T_1}{T_2} = \dfrac{P_1V_1n_2}{P_2V_2n_1} = \dfrac{P_1V_1M_2w_1}{P_2V_2M_1w_2}$

(c) $PV = \dfrac{w}{M}RT$, so $w = \dfrac{PVM}{RT}$

9.3 (a) $h = \dfrac{1}{2}g\left(\dfrac{v}{g}\right)^2 = \dfrac{1}{2}\dfrac{v^2}{g}$

(b) $\text{PE} = mg\left(\dfrac{1}{2}gt^2\right) = \dfrac{1}{2}mg^2t^2$

(c) $\text{PE} = mg\left(\dfrac{1}{2}\dfrac{v^2}{g}\right) = \dfrac{1}{2}mv^2$

It is the same.

9.4 (a) $\text{KE} = \dfrac{1}{2}Nmv^2$ (b) $\text{KE} = \dfrac{3}{2}RT$

9.5 $\dfrac{[Cl^-][Ag(NH_3)_2^+]}{[NH_3]^2} = \dfrac{K_{sp}}{K_i}$

9.6 $\dfrac{[Ni^{2+}]}{[Cd^{2+}]} = \dfrac{1.4 \times 10^{-24}}{3.6 \times 10^{-29}} = 3.9 \times 10^4$

9.7 (a) $x = 5$, $y = 4$ (b) $x = 11$, $y = 6.0$
(c) $x = 5.5$, $y = 6.0$ (d) $x = 9$, $y = 3$
(e) $x = 2\dfrac{1}{3}$, $y = -\dfrac{1}{3}$ (f) $x = 4$, $y = 5$

9.8 (a) $c = 10$ (b) $e = 3 \times 10^{-6}$
(c) $[Ni^{2+}] = 1.2 \times 10^{-3}$
(d) $[Ni^{2+}] = 1.2 \times 10^{-17}$
(e) $[Cd^{2+}] = 1.2$

9.9 (a) ± 6.0 (b) $\pm 10^{-1}$ (c) $[H^+] = 10^{-1}$
(d) $[OH^-] = 10^{-4}$ (e) $[I^-] = 10^{-2}$
(f) -2 (g) ± 1.20
(h) $[Ag^+] = 10^{-15}$

9.10 (a) 2, 3 (b) 9, -1 (c) -4.4, $+3.4$
(d) -5, $+9$ (e) 0.2, 1.5 (f) 0.5, -3.6

9.11 (a) 2.0×10^{-6} (b) 1.4×10^{-7}

9.12 1.9×10^{-3}

9.13 There are straight lines for parts (a) and (b), parabolas for (c) and (d), and a circle centered on the origin for (e). Remember that the same value of x^2 arises from both $+x$ and $-x$.

CHAPTER 10

10.1 (a) $\dfrac{2}{10}$ and $\dfrac{3}{10}$ (b) $\dfrac{8}{2x}$ and $\dfrac{ax}{2x}$

(c) $\dfrac{ay}{xy}$ and $\dfrac{bx}{xy}$ (d) $\dfrac{15}{3}$ and $\dfrac{2}{3}$

(e) $\dfrac{B}{AB}$ and $\dfrac{A}{AB}$ (f) $\dfrac{AB}{B}$ and $\dfrac{1}{B}$

10.2 (a) $\dfrac{5}{10}$ (b) $\dfrac{8 + ax}{2x}$

(c) $\dfrac{ay - bx}{xy}$ (d) $\dfrac{13}{3}$

(e) $\dfrac{B - A}{AB}$ (f) $\dfrac{AB + 1}{B}$

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